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KLEMS productivity accounting for the Polish economy

DARIUSZ KOTLEWSKI

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For my wife Anna

Preface

The *KLEMS productivity accounting for the Polish economy* monograph is devoted to the methodology of decomposing economic growth into factor contributions. The publication, summarising the long-term work of Dariusz Kotlewski, PhD, in economic growth accounting, constitutes a genuine contribution of Polish statistics to this area of research. It is an innovative contribution, as the author does not limit himself to adapting the internationally-established methodology, but incorporates considerable elements of his own work into it.

The methodology of KLEMS productivity accounting is oriented towards identifying the sources of economic growth. For this purpose, an extensive set of statistical data is used, which, on the one hand, makes this kind of accounting very demanding in terms of the scope of data necessary for its implementation, but on the other, contributes to its high level of objectivity, because the results have a solid empirical basis.

The methodology presented in the monograph is not only theoretical, but also applicative. It has been used to perform the relevant calculations. Their results can be found on a data CD attached to the paper version of this publication and on the website of Statistics Poland.

KLEMS productivity accounting is used mainly in studying economic processes and creating a quantitative basis for their further analysis, so it is a valuable tool for economic researchers. Thanks to its explanatory function, it allows the formulation of economic recommendations, forecasts and scenarios of future economic development. It can also be applied to studying the regional economy, the effects of the implementation of economic policy instruments, and the international competitiveness of a country. KLEMS productivity accounting also remains a valuable tool for observing the effects of macroeconomic shocks of a non-economic origin.

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I am especially grateful to Mirosław Błażej, head of the Macroeconomic Studies and Finance Department of Statistics Poland, who was personally involved in many phases of the development of KLEMS productivity accounting for the Polish economy. Thanks to his patience, generosity and substantive support, I was able to attain the competences necessary to prepare the present work. I would also like to thank Krystyna Strzelecka, deputy head of the above-mentioned Department, and all its staff members for their help, kindness and friendly attitude toward the project, on a day-to-day basis.

Last but not least, I would like to express special thanks to all those not mentioned here, particularly to all the involved members of staff of Statistics Poland, whose every contribution proved invaluable.

The author

Contents

List of abbreviations

BAEL	– Polish equivalent of the Labour Force Survey (Pol. Badanie Aktywności Ekonomicznej Ludności)
BDL	– Local Data Bank of Statistics Poland (Pol. Bank Danych Lokalnych)
CPI	– Consumer Price Index
CT	– Communications Technology
ESA	– European System of Accounts (European equivalent of SNA)
GDP	– Gross Domestic Product
GVA	– Gross Value Added
HICP	– Harmonised Index of Consumer Prices
ICT	– Information and Communications Technology
ILO	– International Labour Organization
IMF	– International Monetary Fund
IOT	– input-output tables
ISIC	– International Standard Industrial Classification of All Economic Activities
IT	– Information Technology
MFP	– multifactor productivity
NACE	– Statistical classification of economic activities in the European Community (Fr. Nomenclature statistique des activités économiques dans la Communauté européenne)
NBP	– National Bank of Poland (Pol. Narodowy Bank Polski)
NPV	– net present value
OECD	– Organisation for Economic Co-operation and Development
PBSSP	– Statistical Survey Programme of Official Statistics (Pol. Program Badań Statystycznych Statystyki Publicznej)
PKD	– Polish equivalent of NACE (Pol. Polska Klasyfikacja Działalności)
R&D	– Research and Development activity
SITC	– Standard International Trade Classification
SNA	– System of National Accounts
SUT	– supply-and-use tables
TAOP	– Technical Assistance Operation Programme
TFP	– total factor productivity
TT	– transmission tables

Introduction

In KLEMS economic productivity relative growth accounting (further referred to as KLEMS productivity accounting), the acronym 'KLEMS' consists of the letter symbols customarily used to designate economic variables, which are at the same time the first letters of the following English words: K – *Capital*, L – *Labour*, E – *Energy*, M – *Materials*, S – *Services*. They refer to the production factors used in this kind of productivity accounting. These are basic production factors, sometimes called primary factors, i.e. capital and labour, and the components of intermediate consumption (otherwise referred to as intermediate inputs), i.e. energy, materials and services.

KLEMS productivity accounting is a part of the domain of the economic growth accounting, and essentially involves performing a decomposition of the relative growth of a chosen economic variable reflecting the level of economic activity. For methodological reasons, this variable usually represents gross output or gross value added (GVA), and its relative growth decomposition is done into the contributions of the above-mentioned production factors (and possibly their sub-factors) to this growth.

KLEMS productivity accounting was inspired by an earlier idea of decomposition proposed by Solow (1957), which involves the decomposition of the relative growth of the gross domestic product (GDP) – a variable very closely related to GVA – into labour and capital contributions. Therefore, KLEMS productivity accounting originates from the economic growth theory developed by Solow (1956).

KLEMS productivity accounting is, however, a far-reaching development of Solow's decomposition. New variants (measures) of production factors have been introduced on the basis of their new theoretical concepts; more specifically, the 'services' of production factors replaced the 'stocks' of production factors used in Solow's decomposition. A more in-depth use of available statistical data was made possible, including the above-mentioned economic growth decomposition performed at industry level, e.g. at section and division levels of the Standard International Trade Classification (SITC), the Nomenclature statistique des Activités économiques dans la Communauté Européenne (NACE), or the Polska Klasyfikacja Działalności (PKD) classification systems (the latter being the equivalent of NACE for the Polish economy). It was the contribution of Dale Jorgenson and his associates that proved essential in its development.

A specific feature of any economic growth decomposition accounting is that a certain residual value is obtained, calculated as the difference between the remaining values. In Solow's decomposition, this value is called Solow's residual, and it represents the contribution of an unknown production factor called total factor productivity (TFP). It is similar in the case of KLEMS productivity accounting (and likewise in the productivity accounting performed by the Organisation for Economic CO-operation and Development – OECD), but there the above-mentioned Solow's residual takes the form of a contribution of multifactor productivity (MFP). The MFP value can be considered as a different residual productivity variant of the more traditional TFP. Similarly to TFP, however, deriving it directly from the Cobb-Douglas function indicates clearly that it is the ratio between the considered value for the output (e.g. gross output or GVA) and the inputs in the form of the analysed production factors. This issue will be described in detail further in this study.

Worldwide, the results of the calculations performed in the framework of KLEMS productivity accounting are presented on the World KLEMS internet platform, which in its final form is to consist of regional platforms presenting KLEMS-type accounting such as Asia KLEMS, LA (Latin America) KLEMS or EU KLEMS. Nevertheless, the project is still far from being fully implemented. The most developed regional platform of KLEMS productivity accounting is the EU KLEMS internet site, where the results of decomposition calculations have been presented for a relatively long time. The website presents the results for selected Western European countries, while for the remaining EU member states, including Poland, and countries associated with the EU, the presented results are incomplete. More specifically, they lack the most essential part of KLEMS productivity accounting – the decomposition of GVA relative growth into factor contributions to this growth. Since the EU KLEMS platform is relatively advanced in comparison to other regional platforms, it often contains the results of KLEMS productivity accounting (i.e. calculated by means of the EU KLEMS methodology) for the United States of America and sometimes for Japan (both of which also perform KLEMS productivity accounting according to their own local methodological variants).

The growth decomposition for Poland was performed in the presently outdated EU KLEMS release from 2007 (based on the NACE Revision 1.1 classification system, which is the equivalent of the Polish 'PKD 2004' system). The above-mentioned release adopted relatively far-reaching data imputation methods and did not involve the decomposition of the capital services contribution into sub-contributions of the Information and Communications Technology (ICT) capital services (i.e. capital consisting of computer equipment, communication equipment and software) and non-ICT capital services (i.e. the remaining capital). However, in the later EU KLEMS releases, growth decomposition was no longer performed for the Polish economy (nor for several other European economies). As a result, these decomposition results have lost a substantial part of their analytical value, since the non-updated data are no longer consistent with the updated System of National Accounts (SNA), or its European equivalent – the European System of Accounts (ESA). Moreover, further data revisions for the 1995–2003 period or the SNA revision have not been implemented in the EU KLEMS release from 2007.

The major reason for not performing a decomposition of GVA relative growth for Poland on the EU KLEMS platform is data scarcity – experts and institutions working for the EU KLEMS consortium do not have sufficient access to data. This obstacle results from a number of factors, including those relating to Eurostat's data transmission rules. The scope of data that has to be obligatorily submitted to Eurostat by member countries is insufficient to perform the above-mentioned growth decomposition within the KLEMS productivity accounting framework.

An even more important reason for data insufficiency, however, is the lack of readily available input data in Statistics Poland's data repositories, necessary for performing calculations inherent in KLEMS productivity accounting. In fact, numerous countries struggle with the same issue and as a result they lack growth decomposition on the EU KLEMS main platform.[1] This is caused by the fact that KLEMS productivity accounting is performed not only at aggregate economic levels, but also at industry levels, according to the above-mentioned classifications, including the Polish PKD equivalent. Therefore, highly-detailed data are required, which are often diffucult to obtain.

Work similar to that done in the framework of the EU KLEMS 2007 release was also performed – as an outstanding exception – under the auspices of the National Bank of Poland (NBP). This work involved economic growth decomposition (Gradzewicz et al., 2014, 2018), done, however, at an aggregate level, with no decomposition at the NACE section and division levels, in contrast to the KLEMS productivity accounting performed by Statistics Poland. Moreover, the latter has been continuously updated, whereas the former was an isolated, one-off initiative.

The basic methodological work enabling the performance of KLEMS productivity accounting for the Polish economy involves, besides the implementation of the innovative methodology (developed mainly by US researchers led by Jorgenson) on a national scale, finding appropriate answer to the problems related to the Polish data availability specificities. Moreover, the methodology and practice of KLEMS productivity accounting can be developed outside the adopted international standard which, by necessity, was designed in such a way as to fit as many countries as possible. On the other hand, the interpretation of the results requires a certain

[1] However, in the EU KLEMS 2019 release prepared by Stehrer et al. (2019), only few European countries are left with no decomposition. At the same time, in the EU KLEMS 2019 release the methodology is focused on intangibles. This last release is not yet available on the main EU KLEMS website as of the publishing of this monograph (2021).

degree of knowledge of the economy of a given country. This means that the scope of KLEMS productivity accounting leaves room for various scientific deliberations, in addition to those connected to the practical implementation of this type of accounting in the country statistics.

This monograph is the effect of research work on KLEMS productivity accounting of many years, which has been and continues to be performed under the auspices of Statistics Poland. The first chapter presents the methodology of KLEMS productivity accounting and refers to those elements of the economic growth theory which relate to it. The process of implementing KLEMS productivity accounting in the Polish conditions and innovative activities connected to its development are described in the second chapter. The third chapter is devoted to the most important innovative development of KLEMS productivity accounting, which is a decomposition performed according to the administrative division of the country, i.e. by voivodship, in addition to the typical decomposition carried out at the aggregate and sectoral levels. The fourth chapter presents full KLEMS productivity accounting performed both at the aggregate level for the national economy and at the voivodship level. Due to its specificity, it serves as a bridge between the purely macroeconomic considerations and those related to regional science (as understood by Isard (e.g. 1960) in his quantitative approach).

This monograph is devoted to the methodology of KLEMS productivity accounting and other decomposition accounts strictly connected to it and performed parallel to it during the research process, and to the methodological developments of KLEMS. Purely analytical issues concerning the Polish economy were elaborated on only to the extent required by the main methodological discussion, underlining its importance to the economic analyses. An in-depth description of these issues would require a separate publication.

Chapter 1
Theoretical and methodological bases of KLEMS productivity accounting

This chapter presents the theory and methodological solutions concerning KLEMS productivity accounting, which are or should be applied universally by all countries performing this kind of accounting. As far as deemed necessary, references are made as to the way it is carried out in Poland, although any specific issues related to the implementation of KLEMS productivity accounting for the Polish economy are generally addressed in the next chapter. The thematic scope of the chapter reflects the belief that it is necessary to be acquainted with the universal basics of KLEMS productivity accounting before addressing any further issues. As this kind of accounting stems from the economic growth theory in Solow's version, its important part, Solow's decomposition, is presented here. However, it was the research conducted by Jorgenson and his associates that played the most important role in the development of this kind of accounting; therefore this chapter refers, to the largest extent, to the works carried out or supervised by Jorgenson. The chapter focuses particularly on the relevant formulae adopted by Jorgenson. The way production factors are included in productivity accounts, also in KLEMS productivity accounting, is comprehensively demonstrated.

1.1. Theory behind KLEMS productivity accounting

This part of the study presents the initial idea of decomposition, originating mainly from Solow, followed by a discussion on the problems connected with decomposition and the propositions of their theoretical solutions.

1.1.1. The idea of economic growth decomposition

The history of the measurement and analysis of the growth of economic productivity, or of economic growth, as understood in KLEMS productivity accounting, dates back to the interwar period of the 20th century. At first, the rate of economic growth was believed to have been related to one production factor only, i.e. capital, as in the

Harrod-Domar model (Domar, 1946; Harrod, 1939). However, as early as in the 1920s, the Cobb-Douglas function was tested on statistical data as a production function, with two explanatory variables representing two distinct production factors – capital (K) and labour (L) – in the following form:

$$Y = AK^{\alpha}L^{\beta}, \tag{1}$$

where Y is gross domestic product (GDP) (or any other variable serving as a measure of the level of economic activity, of which gross value added (GVA)[3] is most commonly used), K designates the value of the accumulated capital, L the workload in hours, α the share of capital remuneration in production factors' total income, and β the analogous share of labour compensation. A is the coefficient of proportionality linking the values from the right-hand side of equation (1) with its left-hand side – as the ratio between the result of economic activity (Y) and the inputs of capital (K) and labour (L), it is identified as TFP.

In the following years, the order in which the production factors were cited in accounts based on the general formula (1) often changed, and the symbol α was related to the larger extent to the labour production factor, which usually has a much greater share in the total income of production factors, than to the capital factor; it is so at least at the aggregate economic level, despite the fact that since the onset of the industrial revolution, the role of the capital factor has increased considerably. Moreover, some studies consider the labour factor as primary to the capital factor, and the latter as a cumulative (compound) effect of labour.[4] Therefore, the more recent tendency to attribute the symbol α to the parameter relating to the labour factor seems reasonable.

Essentially, what is more important are the conditions which must be fulfilled in order to assign the values (in the form of respective shares in the joint income of productin factors) to parameters α and β. In theory, these parameters are equal to the above-mentioned shares only when the economy is operating on the basis of perfect competition, which, as we all know, is an approximation only. In fact, though, it is enough if at least one of the aspects of the perfect competition manifests fully, namely if the production factors are remunerated according to their marginal participation in the creation of the product (most often understood as GVA). This

[3] In the present work, the exact definitions of these variables used by Statistics Poland, conforming to the international standards arising from the SNA (SNA'93, or later on SNA 2008), or from its European version (ESA'95, or later on ESA 2010), were adopted.

[4] The 'land' production factor is usually not extracted in productivity accounting; therefore, no reference to this factor is made in the present work. The two basic production factors, i.e. labour and capital, are considered by many authors, including Hulten (2009), as primary. The same approach has been adopted in this monograph, because of these authors' interest in productivity accounting, although this issue might as well be treated as a lexical problem.

means that no asymmetric bargaining occurs, inducing market-power-like effects between the production factors (e.g. between capital owners and trade unions) which would not compensate for each other in the long run and on macroeconomic scales, and which would otherwise lead to a suboptimal allocation of the total income (joined factors' remuneration) between the production factors.

According to the economic theory, particularly the equilibrium approach, maintaining an asymmetry in the economy between the production factors' incomes, involving an inconsistency between their remuneration and the level of their participation in generating the final total product, is impossible in the long run. In productivity accounting, including KLEMS productivity accounting, it is sufficient if this consistency is relative, meaning that the ratio between the labour remuneration and the capital remuneration remains in the same proportion as the ratio between the marginal labour share and the marginal capital share in the marginal total product.

It is important that equation (1) continues to be a good representation of the economic categories used in it, and may further serve for the study of economic processes, also after undergoing relevant transformations. The above would not be possible only thanks to the satisfaction of the equation, because it is always possible to find such a parameter β for parameter α that would cause equation (1) to be satisfied for empirical variables related to factors L and K. This condition is even easier to be met thanks to the fact that variable A is not known empirically, but calculated residually when solving equation (1). If parameters α and β were to be shaped randomly, barely fulfilling equation (1), then the unknown variable A, representing 'productivity' in the theoretical concept, could take on a wide range of possible values, and as a consequence it would not represent any essential economic category. This would undermine any reasoning behind productivity accounting and conducting any analytical research based on it.

The occurrence of constant returns to scale should be assumed here as well, meaning that parameters α and β sum up to unity: $\alpha + \beta = 1$, which results from the principle of replication. This principle assumes that the effects of scales are unlikely to occur in the case of an aggregate economy, as it is theoretically assumed that the growth of production results only from an increase in the number of production plants, not from their expansion – and it is only with the latter process that the increasing returns to scale could take place at the level of individual production plants which would cumulate macroeconomically.

Even if some effects of returns to scale connected with the expansion of production plants do occur, they are compensated for at the aggregate level by a parallel process involving the contraction of some other production plants or even their decline and liquidation, followed by their replacement by some new ones (it can be assumed that for the latter, the negative returns to scale effects are relatively much greater than the positive ones for growing production plants). The potential inaccuracy arising from this assumption is considered much less significant than that arising from the insurmountably imprecise methodology (at least until the publication of this monograph) of evaluating the share of capital remuneration in GVA, based on direct empirical data. Therefore, this share is calculated from formula $\beta = 1 - \alpha,$ where, according to the notation most frequently applied nowadays, β stands for the capital share.

This last assumption is, however, controversial, therefore some decomposition methodologies assume the possible (albeit limited in scope) occurence of certain macroeconomic return-to-scale effects, and as a consequence assume that $\alpha + \beta \approx 1.$ The OECD adopted a methodology assuming that constant returns to scale are in fact only an approximation; therefore, the shares of α and β sum up to unity (as necessitated by technical requirements), but they are defined in a different way (as shares in the sum of the observed production factors' remuneration, which, by definition, is equal to unity or 100%). This means that their summation to unity is not equivalent to the exact occurrence of constant returns to scale in the economy (OECD, 2013, pp. 66–70, 2015, pp. 67–71, 2017, pp. 97–101, 2019, pp. 122–127), because they are not shares in the GVA, as in the case of the KLEMS methodology.

The discussion above suggests that GDP or GVA appearing on the left-hand side of the equation is in fact equalised with the sum of production factors' remuneration (incomes) from the right-hand side, which, following Hulten (2009, pp. 3–5), can be considered quite a strong assumption. However, production function (1) has stood the test of time. Presently, it forms the basis of not only scientific considerations of the neoclassical provenance, but it is also adopted by those of the Keynesian provenance (e.g. Economic and Financial Affairs [ECFIN], 2002, 2006, 2010, 2014).

Regardless of the reflections above, according to the theory devised by Solow (1956, 1957), the following formula is fulfilled for equation (1):

$$\alpha = r\frac{K}{Y}, \tag{2}$$

where r is the weighted average interest rate, according to which the capital is remunerated in the economy, and also the following equation:

$$\beta = w\frac{L}{Y}, \tag{3}$$

where w is the weighted average hourly wage in the economy, and the other variables in these equations have already been described. Obviously, according to notations most often applied nowadays, the symbols α and β in equations (2) and (3) should be swapped. Equations (2) and (3) clearly prove Hulten (2009, pp. 3–5) right.

From equation (1) Solow (1956) derived an equation linking economic growth – $\Delta Y/Y$, understood as the relative (percentage) growth of GDP (in later methodologies – as the relative or percentage growth of GVA), with the contributions of the two resources of production factors, i.e. capital and labour, called Solow's decomposition (1957):

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1 - \alpha) \frac{\Delta L}{L}. \tag{4}$$

In this equation a residual value appears – $\Delta A/A$, called Solow's residual, which represents the input of some 'unknown' factor A (or possibly several 'unknown' factors hiding behind the symbol A) contributing to economic growth, different from the two above-mentioned basic ones.[5] This 'unknown' factor A (or aggregate of factors) is total factor productivity (TFP), often called simply 'productivity'. Based on equation (1), it can be quickly described as the ratio of the produced output to the inputs of production factors.[6]

According to Solow, the above-mentioned residual represents technological progress, in this model understood as exogenous, which accompanies the growth of capital and labour inputs. For this reason Solow's theory is often referred to as the theory of exogenous growth, in contrast to the endogenous growth theory variants developed later. In exogenous growth models, the elements of technological and organisational progress (the former being far more important) appear in the economy on a *deus ex machina* basis, i.e. spontaneously and independently from the condition of the economy.

1.1.2. Issues related to economic growth decomposition

The concept of the long run economic growth generated exogenously started to raise certain doubts, which led to the emergence of endogenus growth models. In these models, technological progress resulted from processes occurring in the economy, mainly from outlays on research and development (R&D) and on human capital and knowledge capital development, which was accompanied by positive external effects, also of a spill-over character. Therefore, it was thought that in times

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[5] The joint value of weighted quantity growths of production factors is different from the value of GDP growth, which is equivalent to the statement that the joint value of weighted aggregate price growths of production factors is different from the aggregate price growth of GDP. See: Hulten (2009, p. 17).

[6] Technically, equation (4) is representative for the 'discrete approach' (where the delta symbol, Δ, is used), where measurable time intervals occur, usually annually. Formula (4) in continuous time takes the form of: $\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \alpha \frac{\dot{K}}{K} + (1 - \alpha) \frac{\dot{L}}{L}$, where the dot over the given variable indicates its time derivative. From the point of view of the present theoretical presentation and the empirical study which it concerns, it is of no importance whether the formulae is recorded in a discrete or continuous form. Therefore, in the remaining part of the paper, only the discrete form will be used.

when the technological progress would result mainly from deliberate outlays on development, endogenous growth models would be more effective in analyses of the economy than the exogenous ones.

However, such phenomena as the information revolution started without any outlays (in private garages of some enthusiasts in California), and only later did large firms start to deliberately invest in ICT. However, although such investments are discounted to a large extent in the capital factor contribution to growth. Moreover, many economies of the emerging markets develop much faster than developed economies, often without large investments into the development of technology, which is often just imitated, and therefore of an exogenous origin in the light of not only theory, but also empirical facts. At the same time, the way the contribution of this progress is presented – as a variable calculated residually – indicates that it includes all possible components of this progress as their conglomerate, regardless of whether they are of an exogenous or endogenous origin.

Therefore, despite numerous attempts to construct endogenous growth models (see: Barro & Sala-i-Martin, 2003; Lucas, 1988; Rebelo, 1991; Romer D., 2011; Romer P. M., 1994), and despite their cognitive value, the model originating from Solow's thought has persisted in economic growth accounting (without any major changes to its core). Additionally, for example Krugman (2013) demonstrates that endogenous growth models do not have any advantage over the basic exogenous growth model devised by Solow, i.e. are not more empirically efficient, because they rely on too many assumptions relating to unmeasurable values.

An additional interpretation of Solow's residual is that the joint capital and labour use result in a greater productivity growth in the economy than when used separately. In other words, a synergy exists between them. Presently, the common belief is that this residual is mostly about the technological and organisational progress disembodied in labour and capital. Since Solow's residual is calculated residually as the difference between the other values (which also means that equation (4) is always met in the calculus practice), it can also contain all the other contributions to economic growth, not defined by the model, other than technological and organisational progress (if any of them ever exists),[7] and all kinds of tool deviations.[8]

Although equation (4) is always met, it remains only an approximated equivalent of equation (1), as Solow introduced some mathematical simplifications during its formulation. For small changes, indicated with the Δ symbol in equation (4), it is

[7] It can be ascertained that such additional factors do exist although they cannot be easily approached model-wise. These are e.g. social behaviours together with accumulated historical artefacts, the consequences of these behaviours in the form of material and symbolic culture. That is why the productivity of growth factors differs significantly across regions and countries of the world. The environmental limitations should be considered here as well. See e.g. Brdulak (2012).

[8] The standard components of TFP are labour and capital efficiencies, and labour and capital capacity use (ECFIN, 2014, pp. 9–10).

theoretically convergent with equation (1). However, if the changes are large, the equation is still met, but no longer equivalent to equation (1), as the solved value of A for the two equations is somewhat different. In order for equations (1) and (4) to produce the same solution for unknown variable A, Solow's decomposition should take the form of a trans-log equation, which is at present regularly applied in economic growth accounting (further information on this issue is provided in part 1.2).

The ongoing discussion among researchers (e.g. Hagemann, 2009; Sato, 1964; Scarfe, 1977) has led to the conclusion that Solow's model is clearly superior to the Harrod-Domar model. In result, the latter model is no longer developed as extensively as the decomposition devised by Solow. The Cobb-Douglas production function and Solow's decomposition became the basis for the development of economic growth accounting within the supply-side theory of economics, and as such, also the basic source of inspiration for the development of KLEMS growth accounting.

The numerous doubts related to the interpretation of Solow's residual (or to be more exact – TFP[9]) have led to the development of several new versions of multifactor productivity growth concepts.[10] In result, other production factors were introduced or the traditional production factors – capital and labour – were divided into sub-factors. These new factors and sub-factors were not always entirely separate from each other; in consequence, some parts of growths of these factors replicated and were multiplied by more than one share from their series, noted usually as α, β, γ, etc. or as ν with relevant subscripts, as in the EU KLEMS manual (Timmer et al., 2007a) and numerous other references.

Adding such overlapping factors resulted in an unsubstantiated reduction of the role of Solow's residual in the production function. Essentially, the production factors present in the production function should be entirely separated to ensure that the calculations assessing the contribution of TFP are correct.[11] This also concerns the variant of Solow's residual, which is applied in KLEMS productivity accounting (and also in OECD productivity accounting), i.e. the contribution of MFP.

In the further development of the theory, it was observed that the decomposition of the GVA relative growth (and therefore also the possible application of the decomposition of GDP relative growth) is not neutral in relation to some possibility that a deviating impact is exerted on the calculated TFP or MFP. This deviating impact involves the occurrence of a substitution phenomenon between the contributions of labour and capital factors, and the intermediate inputs (intermediate consumption) contribution to the gross output relative growth. The growth of the

[9] TFP can be considered as an unknown 'production factor' designated by A in production function (1) or Solow's decomposition (4), whereas Solow's residual is its contribution designated by $\Delta A/A$.

[10] It is a MFP concept because there are at least two factors – capital and labour (OECD, 2001, pp. 12–18).

[11] However, this does not mean that theoretically it is completely impossible to extract certain sub-contributions of some factors included in TFP.

labour factor can be substituted by e.g. the growth of external services acquired through outsourcing. Instead of employing additional staff, firms can outsource services to external companies. Such a situation involves the labour factor growing slower, while the intermediate consumption grows faster. On the other hand, the increase of certain tangible capital, often consisting of small-value elements, can at times be included in the intermediate inputs growth instead of being included in tangible capital growth and vice versa, depending on the regulations or accounting practice rather than the needs of the economic growth theory. Nevertheless, in this case leasing proves most important as it causes some elements included as capital to become in fact part of intermediate consumption as the purchase of external services.[12]

There can be more situations involving the substitution of the main production factors contributions by the contribution of intermediate consumption to gross output relative growth. For instance, capital created as a result of a large investment outlay with low running costs can be replaced by capital created as a result of a small investment outlay with high running costs and vice versa. The decomposition of the GVA relative growth may achieve the same result for TFP and MFP[13] relative growths (as in the case of the decomposition of gross output relative growth) only if, besides TFP or MFP, the technological and organisational progress is embodied only in the two primary factors, i.e. in capital and labour, and if they do not relate to intermediate consumption.

However, the assumption involving the absence of technological or organisational progress, i.e. changes in the productivity of intermediate consumption, treated as a specific factor of production in addition to the two primary ones – labour and capital, is untenable, especially in a long-run period, which may entail, e.g. certain decreases in material intensity, and especially energy intensity in the economy. Such decreases *ceteris paribus* lead to a contraction in the contribution of intermediate consumption.

These potential changes in the contribution of intermediate consumption are accompanied by related changes in the contributions of the basic production factors, which altogether deviates the value of the TFP or MFP contributions if they are calculated residually only from the decomposition of the relative growth of GVA. Thus, in such a case a difference may arise between TFP or MFP calculated from the decomposition of the relative GVA growth and TFP or MFP calculated from

[12] Attempts are made to modify the capital contribution in order to include leasing services, which is a separate issue (see e.g. de Haan et al., 2005); it would, however, require a reform of the SNA (Timmer et al., 2007a, p. 42). Leasing firms have their own capital and their services are part of the GDP, therefore, raising no problems at the aggregate level, but at the same time the distribution of capital between sectors changes, which affects the decomposition accounting done at industry levels.

[13] Strictly speaking, not the same but convertible results. In order not to complicate this discussion here, this issue is referred to in a subsequent part of the study.

the decomposition of the relative gross output growth, even when they are mutually convertible. Since this issue is complex and addressing it provides valuable additional insight into economic processes, it is considered in more detail in Chapter 2.

A change in the contribution of intermediate consumption may also occur, involving its increase resulting from a continuous rise in the prices of certain raw materials, especially energy carriers, in addition to the increasing use of outsourcing in the economy, and other transformations. These changes do not necessarily translate entirely into the volume of gross output, but may affect the size of the residually calculated MFP. This is especially true with respect to the dynamics of these changes. As a result, when a noticeably high rate of change in intermediate consumption is observed, a discrepancy may also appear between the MFP growth calculated residually from the decomposition of gross output growth and the MFP growth calculated residually from the decomposition of GVA growth – also because changes in the economy do not spread fast enough for markets to clear.

The answer to this problem is to perform a decomposition of gross output relative growth instead of a decomposition of the GVA relative growth, which requires introducing intermediate consumption as an extra factor in the production function. In addition to the components of the GVA growth decomposition, the contribution of intermediate consumption X should be included in this case, as an additional component of the decomposition of gross output Y_{GO}. The production function in this case should take the general form:

$$Y_{GO} = AK^{\alpha}L^{\beta}X^{\gamma}. \tag{5}$$

For this reason, equation (4) for the decomposition of economic growth should be modified so that changes in factor contributions are offset by changes in intermediate consumption contribution, the latter occurring in the opposite direction. Thus, the substitution effect ceases to affect Solow's residual (understood as TFP or MFP contribution). Also for this reason, instead of the previously performed decomposition of the relative GDP growth, as in Solow's original model, it is preferable to perform the decomposition of the relative GVA growth, which is methodologically consistent with the decomposition of the relative growth of gross output, for which in national accounts (based on SNA or ESA) the following equation is used:

$$Y_{GO} = X + Y_{GVA}, \tag{6}$$

where Y_{GO} – gross output, X – intermediate consumption, Y_{GVA} – gross value added.

These changes and further modifications, whose main authors are Jorgenson and Griliches (1967) and Jorgenson et al. (1987, 2005) (more on this in subsection 1.2), were accompanied by a conceptual integration with the statistical apparatus based on SNA or ESA, therefore a sub-decomposition into statistically considered sectors, i.e. into NACE[14] sections or divisions, also became possible. These systems were inspired by Leontief's (1966) ideas in the form of input-output tables (IOT), or supply-and-use tables (SUT). Leontief's concepts were presented in a more recent version and more closely related to KLEMS productivity accounting (since the cited author also deals directly with this kind of accounting), for instance by Timmer (2012).[15] This is of a decisive importance for the superiority of KLEMS productivity accounting (as well as the OECD productivity accounting) over other such decompositions derived from Solow's decomposition model (1957), often performed in an unsystematic way and without the formal discipline resulting from the SNA and ESA used in official statistics.[16]

This is firstly because the analysis at the level of appropriately defined sectors of the economy has proven much more promising in explaining the sources of economic growth. In different sectors by NACE, the factor contributions and TFP or MFP contributions to economic growth can be quite different. Thus, comparisons between various countries at the sectors (or industries) level of the economy can become a much more valuable source of information than comparisons made only at the level of whole-economy aggregates. Secondly, the phenomenon of reallocation of labour and capital across sectors becomes clearly conspicuous.

The ability to account for differences in productivity between various types of capital and labour is most significant for accounting itself (and the accounting technique). Considering these differences is related to the adoption of such concepts as the contribution of 'labour services' and 'capital services' as representing factor inputs instead of the previously used measures including the contribution of labour resources (labour hours, for instance) and the contribution of capital resources (fixed capital stock), which, incidentally, are still used in some other versions of economic growth decomposition accounting. The latter resources (stocks) are also necessary as they act as input data in the KLEMS (and OECD) productivity accounting and are used to calculate the aforementioned factor services' contributions. Since these services are not directly observable, a description of how the calculation is performed is provided in the further part of this study. A new term was introduced to replace TFP in growth decomposition accounting in connection with the use of factor services categories and their contributions (instead of categories of factor

[14] The PKD 2004 statistical classification, which is the equivalent of NACE 1 (NACE revision 1.1 to be exact) and the PKD 2007 statistical classification, which is the equivalent of NACE 2 (precisely NACE revision 2) is applied in the Polish statistical framework. From the point of view of KLEMS productivity accounting no difference exists between the Polish and European classifications; differences appear at much lower sub-aggregations.

[15] The issue of international input-output flows was also addressed in this work.

[16] In the Polish conditions a similar way of thinking was demonstrated by Sulmicki (1978), who created analogous decision tables (nets) in the field of management.

resources (or stocks) applied before) in the production function and in the decomposition based on it – MFP. This is related to the fact that Solow's residual calculated by means of the methodology designed for its calculation which takes into account the determination of factor services' contributions becomes slightly (sometimes quite significantly) different. MFP can therefore be treated as a later variant of TFP.

Thus, according to the current theoretical findings, KLEMS economic productivity growth accounting should in principle be based on the decomposition of the relative growth of gross output at the level of selected sectors of the economy, in practice at the level of NACE sections and divisions, into the contributions of intermediate consumption (intermediate inputs), factor services and MFP. According to the theoretical framework of KLEMS productivity accounting, this is the most appropriate way to measure the MFP contribution to the economy, because theoretically it is then that MFP most closely corresponds to the idea of technological and organisational progress disembodied in factors.

Despite these theoretical findings, doubts still remain as to the use of gross output as a measure of the level of economic activity. The first and most obvious controversy is that the gross output can sometimes grow only because of the contribution of intermediate consumption, which is not part of the final production output. The increment in gross output is therefore not representative for the increase in the result of economic activity, and its decomposition only theoretically facilitates a more accurate determination of MFP's contribution to this increase.

The second important controversy related to intermediate consumption is that the decomposition of the relative growth of gross output is flawed by the distorting effect of changes occurring in the vertical integration of firms in the economy. The more the firms are vertically integrated in the economy, the larger the part of 'real' intermediate consumption becoming statistically unobservable. This relative growth may then not be representative for the idea of economic growth due to the changes taking place in vertical integration. More importantly, this particularly affects the comparability of results across countries, which vary greatly in terms of the vertical integration of firms. In order to reduce this problem, the economy is divided into particular aggregations, i.e. sectors, consisting of groupings based on the NACE division lines, between which vertical integration, if possible, does not occur or it occurs to a limited extent. However, this problem cannot be solved in a completely satisfactory way.

The common lack of 'deflators' for intermediate consumption in world statistics poses an additional, important technical problem in the implementation of KLEMS productivity accounting, especially if one divides the contribution of intermediate consumption to the relative growth of gross output into three sub-contributions, as practised in KLEMS accounting. These include sub-contributions of the growth in energy consumption, material consumption and external services use (all these quantities presented in value terms, not physical). For instance, deflators for intermediate consumption are now available for Poland, though without this breakdown. Another technical problem is related to the necessity of introducing additional data into decomposition accounting, as the data tends to be of varying and sometimes insufficient quality.

The return to the simpler version of decomposition, i.e. to the decomposition of the relative growth of GVA instead of the previous one seems to be the only known way to avoid the above-mentioned problems connected with the decomposition of the relative growth of gross output;[17] this solution has in fact recently become standard practice.[18] This approach assumes that the problem of distortions in the international comparability of the decomposition accounting results, arising from differences in the vertical integration of firms among countries, is more important than the problem of the substitution between intermediate consumption contribution and factor contributions, although this substitution theoretically and in practice generates the above-mentioned deviations in the calculations relating to TFP or MFP contributions. Solow's residual is in this case interpreted somewhat differently, i.e. not necessarily as technological and organisational progress disembodied in labour and capital, as was originally established, but as the ability of sectors to capture value[19] or otherwise to participate in the income (OECD, 2001, p. 23).

The abandonment of a solution allowing the reduction of the described substitution problem is additionally beneficial in the sense that GVA is close to GDP and even assumed by theoreticians to be identical, if the conventional definitions used by statistical offices, which take into account indirect taxes and subsidies, are disregarded. Thus, the decomposition of the relative growth of GVA is more appealing to the imagination of the user accustomed to GDP being the most widely used measure of the level of economic activity.[20] This approximation is justifiable especially when there are three theoretically possible statistical definitions and therefore also methods for calculating GDP.[21] Hence, in most cases only the decomposition

[17] This rationale is based on Hulten's work (2009, pp. 25–28) and others.

[18] As a result, the experience about gross output decomposition concerns only some countries that have extensive statistics.

[19] This issue is considered by Bowman and Ambrosini (2000) at the microeconomic level.

[20] According to the methodology applied in statistical offices (also in Statistics Poland) GDP is equal to GVA plus taxes on products (mostly VAT) minus subsidies to products.

[21] The expenditure approach assumes that GDP is equal to the expenditure on all final goods produced during the year. On the demand side, GDP is therefore calculated by means of the following formula: GDP = consumption + investment + government expenditure (excluding transfers) + change in stocks (in a closed economy). In the income approach, it is assumed that GDP is equal to the sum of incomes of all factor owners. On the income side, GDP is calculated as follows: GDP = labour income + capital income + government income + depreciation. In the production approach, the value of produced services and final goods is calculated by subtracting from the total production the value of goods and services consumed in that production. According to this approach, GDP is calculated by means of the formula: GDP = gross output – intermediate consumption = GVA. However, statisticians add taxes on products (mainly VAT) to GVA and subtract subsidies to products; therefore, GDP calculated this way is slightly different from GVA.

of the relative growth of GVA is performed, since the decomposition of the relative growth of gross output, although theoretically allowing a better determination of the MFP, fails to provide sufficiently clear information to the user – including a professional one – who is not an expert on this accounting and, therefore, will not fully grasp the significance of this difference, most often inconspicuous in practice and difficult to interpret in the idiographic description of economic processes.

In the light of these uncertainties, it may be considered sensible to perform both the decomposition of gross output growth and the decomposition of GVA growth, as long as the statistics allow it. An attempt can then be made to deliberately examine the difference connected with the residually calculated productivity TFP or MFP (within KLEMS productivity accounting this has been done for MFP, as discussed in Chapter 2) and to interpret this difference.

As a result of this conceptual development, statistics for the economic productivity accounting on the EU KLEMS platform are based on the measurement of the growth of different kinds of labour services (L) and capital services (K) within the decomposition of the GVA growth. In the case of the optional decomposition of the gross output growth done for some countries, the statistics are also based on the measurement of the growth of intermediate consumption or its components such as energy (E), materials (M) and services (S). These measurements, disaggregated into selected sectors created according to the NACE activities (or their equivalent for non-European countries – International Standard Industrial Classification of All Economic Activities, ISIC) i.e. into appropriate section groups, sections, division groups and divisions, and possibly further, provide the foundation for constructing databases, which after appropriate conversions should be used further in the algorithms used in KLEMS productivity accounting.

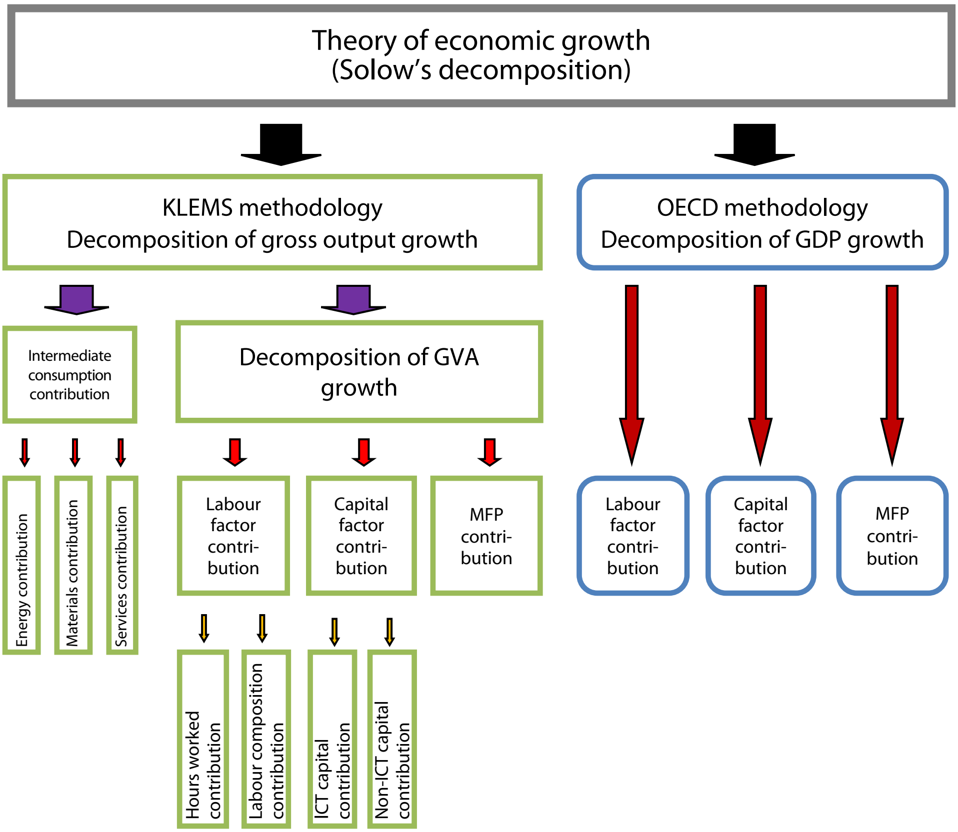
The old version of the accounting on the EU KLEMS platform implemented in the NACE 1 classification system applied a division into 72 sectors, while the new versions implemented in the NACE 2 system use a division into 34 sectors. The greater granularity of the division in the old system, does not mean that it is better, as it may lead to increased inconsistencies between the results of the decomposition of the gross output growth and the results of the decomposition of the GVA growth, resulting from vertical integration of firms; and such inconsistencies would be difficult to interpret theoretically.

The problem of a too deep division into sectors may also occur, because the statistical category of the 'dominant activity' may hide secondary and yet statistically significant activities. In this situation, it is necessary to select such a division into sectors, i.e. such aggregates according to NACE division lines, for which secondary activities occur ideally only within them, and not between them, similarly to the case of vertical integration of firms. This problem also concerns the decomposition of the GVA growth, not only the decomposition of the gross output growth.

In the light of these two situations (the problem of vertical integration and secondary activity), the need for a compromise-based solution for the disaggregation into selected economic sectors becomes evident. Establishing such a solution internationally is challenging, as countries differ in the characteristics relevant to both situations. However, it is currently assumed that regional platforms of the KLEMS accounting, such as EU KLEMS, will have integrated methodologies in this respect. Differences will remain between regions. Countries such as Japan and the United States of America run KLEMS productivity accounting on a two-track basis, meaning that they are the only non-European countries present on the EU KLEMS platform, but at the same time each of them performs this kind of accounting according to their own divisions into sectoral aggregations – these data are to be published on the World KLEMS platform, which is ultimately to be of a global character. This is where the specific choice of sectoral aggregations used in the EU KLEMS productivity accounting comes from. In the KLEMS productivity accounting for the Polish economy presented here, the same division into 34 industries was adopted as in the latest versions of the EU KLEMS platform. The reason behind this choice was to maintain international comparability of decomposition results of KLEMS accounting between the Polish economy and other European countries, for which this accounting is also performed.

The methodology initially based on the same economic growth theory (Solow, 1956), or more precisely, on its emanation in the form of Solow's decomposition (1957), is also used in OECD productivity accounting (see Figure 1). However, this methodology is intended to assure the comparability of economies for the largest possible group of countries, so only the GDP growth rate decomposition is currently implemented instead of the decompositions of gross output or GVA growth rates (OECD, 2001, 2009, and especially: OECD, 2013, pp. 66–70, 2015, pp. 67–71, 2017, pp. 96–101, 2019, pp. 122–127). It completely bypasses any reference to the contribution of intermediate consumption. The decomposition of GDP growth in the OECD methodology, instead of considering the GVA growth decomposition, involves the loosening of some very rigid assumptions, such as constant returns to scale, which are treated as valid in approximation. Also, the labour quality (i.e. the labour composition) contribution is not extracted, as in KLEMS productivity accounting, and the contribution of capital is not divided into sub-contributions of ICT capital and non-ICT capital. However, also here the contributions of factor services are considered, not the contributions of their resources (stocks). Instead, the OECD methodology adopts a more detailed sectoral division, which partly compensates for the lack of the extraction of the contribution of labour quality (labour composition), as narrow sectors differ in this category between one another (see Wölfl & Hajkova, 2007).

Figure 1. Two domineering methodologies in economic growth accounting



Source: author's work.

1.2. Methodology of KLEMS productivity accounting

The main features of the methodology of the European variant of KLEMS productivity accounting were presented in a document prepared for the EU KLEMS consortium (Timmer et al., 2007a), published together with the 2007 online release of the data sets.[22] This document was accompanied by others presenting the methodology of handling input data for calculations performed within KLEMS accounting. This methodology is individualised for particular countries and it also includes the Polish economy (Timmer et al., 2007b, pp. 121–129).[23]

[22] For this year see also Koszerek et al. (2007).
[23] These elaborations were later updated to some extent, see e.g. Gouma and Timmer (2013a, 2013b).

This is related to the fact that within the above-mentioned EU KLEMS release data for the Polish economy were also published, including the most important element of KLEMS productivity growth accounting, i.e. the decomposition of the relative GVA growth into factor services and MFP contributions (although without the decomposition of capital services contribution into ICT and non-ICT capital services sub-contributions). The latter document has already become outdated, as further data releases have been published on the EU KLEMS internet platform, this time without the decomposition of relative GVA growth for the Polish economy. Nevertheless, the methodology presented in the first study remains basically up to date, therefore it can be treated as a kind of introductory manual to the domain of KLEMS productivity accounting.

However, the basic methodology of KLEMS productivity accounting had already been developed by Jorgenson and his associates. The most important studies in this field include those by Jorgenson (1963, 1989), Jorgenson and Griliches (1967) and Jorgenson et al. (1987, 2005). In addition to the aforementioned EU KLEMS 'handbook', the methodology was summarised by O'Mahony and Timmer (2009) for the European EU KLEMS variant of this accounting, and more generally by Jorgenson (2009). The more recent sources which present the KLEMS methodology include Timmer et al. (2010) and Havlik et al. (2012). After the first decade of the 21st century, the KLEMS methodology stabilised (including EU KLEMS) and only recently have some attempts been made to develop it further – also in Poland, as part of the work carried out by Statistics Poland on KLEMS productivity accounting.

However, this issue is relatively new in Poland. It is necessary that the methodology refers to Polish characteristics concerning the availability of input data for the calculations essential in KLEMS productivity accounting. In addition, these input data are not (apart from the above-mentioned 2007 release of EU KLEMS) prepared on the EU KLEMS platform in a way that would allow the decomposition of relative GVA growth into factor contributions and MFP contribution, which is a crucial element of this accounting. For those reasons, separate research work on KLEMS productivity accounting was carried out in Statistics Poland (Kotlewski & Błażej, 2016, 2018, 2020a, 2020b).[24]

[24] The Polish economic literature includes studies on TFP which include methodologies other than KLEMS. See for example: Florczak and Welfe (2000), Próchniak (2019), Rapacki and Próchniak (2012), Tokarski et al. (2005), Welfe (2003). In the work of Tokarski et al. comparisons are made between regions in Poland, which can be considered as a kind of anticipation of the systematic but independent study presented in Chapter 3 of this monograph. The work by Rapacki and Próchniak presents analyses which have only recently become possible to perform in the light of KLEMS productivity accounting, extended by the sectoral dimension (see Kotlewski & Błażej, 2020b). Próchniak's work addresses the problem of competitiveness, which is also developed including the sectoral dimension within KLEMS accounting. The sectoral dimension is an important added value of the KLEMS methodology.

In this section, the methodology of KLEMS productivity accounting will be presented using equations in which, according to the principle adopted in this accounting, relative increments are presented as logarithmic expressions. In the course of the work on KLEMS accounting, calculations were also performed using ordinary increments, both at individual aggregation levels and using the Törnqvist procedure for the aggregation. These calculation methods are summarised in Table 1.

Table 1. Four methods of calculating relative growth rates for aggregate values

Method	Relative growth	Evaluation of the method
Decomposition at all aggregation levels:		
I	$\Delta x/x$	appropriate
II	$\Delta \ln x$	appropriate
Decomposition only at the lowest selected aggregation levels and their aggregation by means of the Törnqvist procedure:		
III	$\Delta x/x$	inappropriate
IV	$\Delta \ln x$	appropriate

Source: author's work.

The Törnqvist procedure is necessary to calculate the contributions of labour services and capital services (according to the methodology developed by a variety of authors, including those mentioned above), which proved particularly important for the labour services category. At the same time, findings showed that the contribution of capital services does not differ significantly from that of the capital itself (more on that matter in the subsequent part of this monograph), at least at the level of higher aggregations, including the aggregate level.

The inappropriate III method produced the most divergent results in some situations, especially when the input data to the accounting contained identified errors. The implementation of the accounting according to the four methods shown above has therefore allowed the systematic tracking of errors, especially in the initial phase of constructing KLEMS productivity accounting, which remains significant since the calculations involved in this kind of accounting are of a particularly complex nature. At the same time, for calculus-related and methodological reasons, some values have to be calculated at all levels of aggregation – this concerns the decomposed values, meaning e.g. the relative GVA growth rates or the relative gross output growth rates, as well as the results for the MFP contribution obtained residually at all levels of aggregation. From this point on, only the MFP associated with KLEMS productivity accounting will be considered in this monograph. Reference to the TFP variant will be made in the further part of this study in connection with the regional decomposition accounting presented in Chapter 3.

1.2.1. Gross output and gross value-added decomposition accounts

The equation for the decomposition of the relative gross output growth in given sector j in period t[25] may be considered as the starting point in the formalised version of KLEMS productivity accounting:

$$\Delta \ln Y_{jt} = \bar{v}_{jt}^{X} \Delta \ln X_{jt} + \bar{v}_{jt}^{K} \Delta \ln K_{jt} + \bar{v}_{jt}^{L} \Delta \ln L_{jt} + \Delta \ln A_{jt}^{Y}, \tag{7}$$

where Y_{jt} is the gross output, $X_{jt}t$ – intermediate consumption, K_{jt} – capital services, L_{jt} – labour services,[26] and A_{jt}^{Y} is the gross-output-based MFP. These values are indexed by subscript which indicate that they refer to sectors j (otherwise called industries), usually understood in KLEMS productivity accounting as groups of NACE sections, NACE sections, groups of NACE divisions or NACE divisions, and periods t. Δ denotes the change in values between period t and $t-1$, which are usually identified as yearly periods. If the changes are minor, which is usually the case for one-year periods, the approximation $\Delta \ln x = \Delta x / x$ is satisfied, i.e. in this case a relative change is involved, e.g. expressed as a percentage. The logarithmic expression is also assumed to be more appropriate for larger changes in aggregate economic quantities observed in multi-year periods (which results from the theoreticians of this accounting adopting the Törnqvist procedure, considered the most appropriate in aggregation), as it suppresses divergences and error accumulations (Diewert, 2004; International Monetary Fund [IMF], 2004; Milana, 2009; Schreyer, 2004).[27]

In turn, $\bar{v}$ with appropriate indices denotes the average share – in value terms – of a given factor (in the superscript defined as intermediate consumption X, capital services K and labour services L) between periods t and $t-1$, which is calculated according to the general formula $\bar{v}_t = (v_t + v_{t-1})/2$ (for simplicity's sake, subscript j present in equation (7) and denoting industries was omitted here), i.e. in this case linear interpolation is performed. The $\bar{v}$ shares themselves (with appropriate indices) are the shares of remuneration values of the basic production factors (labour and capital) and of the value of intermediate consumption in the gross output. In calculus practice, these are assumed to add up to unity, which is related to the aforementioned requirement of perfect competition and constant returns to scale; otherwise the term 'shares' should be replaced by a more general and non-unitising

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[25] The symbols in the formulae are original after Timmer et al. (2007a), who refer to symbols applied earlier in the above-mentioned works of Jorgenson et al. (1967, 1987, 2005).

[26] Very often it is the 'labour' factor that is mentioned before the 'capital' factor, but this is of no significance in the present discussion.

[27] The use of logarithms attenuates the effects of large outlier observations for positive increments, which occur much more frequently than negative increments.

term 'elasticities' (of the response variable in relation to the explanatory variables). These shares satisfy the following equations (O'Mahony & Timmer, 2009, p. F376):

$$v_{jt}^{X} = P_{jt}^{X} X_{jt} / P_{jt}^{Y} Y_{jt}$$

$$v_{jt}^{L} = P_{jt}^{L} L_{jt} / P_{jt}^{Y} Y_{jt}, \tag{8}$$

$$v_{jt}^{K} = P_{jt}^{K} K_{jt} / P_{jt}^{Y} Y_{jt},$$

where P_{jt}^{X}, P_{jt}^{L}, P_{jt}^{K} and P_{jt}^{Y} are the prices of intermediate consumption equivalent units X_{jt}, prices of labour services equivalent units L_{jt}, prices of capital services equivalent units K_{jt} and prices of gross output equivalent units Y_{jt} (when these units are different, they should be understood as weighted average prices), respectively. The subscripts indicate that all quantities refer to sectors by NACE (industries) *j* and to periods *t*, usually yearly.

Before the concepts of labour services and capital services were introduced in growth accounting, the symbol L usually denoted the mobilised labour stock (resource), i.e. the number of physical labour units, e.g. the number of man-hours. However, data on these man-hours are still necessary for the calculation of the labour services contribution(s), with hours actually worked rather than hours paid (or possibly other similar categories). Therefore, the current growth accounting term used for the stock of labour factor is *hours worked*. The consideration of hours worked as an input measure for the calculations is also a response to the problem arising from the simple counting of employed persons. It may disregard the accounting effects arising from part-time work overtime, absenteeism (sick or other) and transfers due to, for example, holidays (especially long ones, such as maternity leave, etc., paid or unpaid), as well as transfers objectively necessitated by tasks currently required in the enterprises.

Prior to the introduction of the concepts of labour services and capital services in growth accounting, the *K* symbol denoted the value of the stock of fixed assets as the most appropriate approximation of the stock of capital involved in the production of goods and services. Data on fixed assets, i.e. the stock of capital, as in the case of the stock of labour, are still necessary to calculate the contribution(s) of capital services, since the flows of the latter are not directly observable. Thus, earlier versions of economic growth decomposition accounting based on Solow's concept used the growth of the capital factor stock category instead of the growth of capital factor services category in the calculations, while in the more recent versions of productivity accounting, including KLEMS productivity accounting, the opposite is practised.

The assumption of constant returns to scale means that the share of intermediate consumption v_{jt}^{X} in gross output can in practice be calculated (and this is done in KLEMS productivity accounting) as the ratio of the value of the difference between the value of gross output and GVA to the value of gross output itself ,[28] and the share of the remuneration of capital v_{jt}^{K} – as the ratio of the value of the difference between GVA and the value of the remuneration of labour to the value of gross output.[29] Thus, only labour remuneration share v_{jt}^{L} is calculated independently as the ratio of the value of the labour remuneration to the value of gross output.[30] If the relative growth rates (in percentages) of these factors of production and of intermediate consumption are multiplied by the aforementioned shares (also in percentages), then the contributions of factors of production and of intermediate consumption to the relative growth of gross output are obtained and expressed in percentage points.

In addition to the previously mentioned practical advantage resulting from the use of logarithms (involving the suppression of some errors), also the theory is in their favour in comparison to ordinary relative-growth expressions. After all, the shape of formula (7) is the result of attributing a trans-logarithmic form to the production function in order to confer an additive character to the decomposition, according to the relative changes in the values of its arguments. Hence, it is the result of a mathematical transformation according to the general formula:

$$Y = AB^{\alpha}C^{\beta}D^{\gamma} \leftrightarrow \ln Y = \ln A + \alpha \ln B + \beta \ln C + \gamma \ln D, \tag{9}$$

in which the symbols used can take any value and represent any quantity, since it is a general mathematical equation. In order to obtain a suitable form that could be applied in economic growth accounting, the equation on the right-hand side of formula (9) still needs to be transformed for the growth rates by inserting the Δ sign, also according to the general rules of mathematics:

$$\begin{gathered}\ln Y = \ln A + \alpha \ln B + \beta \ln C + \gamma \ln D \rightarrow \Delta \ln Y = \\ = \Delta \ln A + \alpha \Delta \ln B + \beta \Delta \ln C + \gamma \Delta \ln D,\end{gathered} \tag{10}$$

except that this time it is a one-way transformation, so the arrow symbolising this transformation is one-sided in formula (10) as opposed to the arrow in formula (9). From the more general equation on the left-hand side of formula (10) a less general equation on the right-hand side can be derived and not the other way around.

[28] Therefore: $v_{jt}^{X} = (GO - GVA)/GO$, where GO is the gross output.
[29] Therefore: $v_{jt}^{K} = (GVA - LR)/GO$, where LR is the remuneration of labour (otherwise labour compensation).
[30] Therefore: $v_{jt}^{L} = LR/GO$.

For the purposes of KLEMS productivity accounting the appropriate specific economic quantities described by equation (7) need to be simply inserted into the general equation on the right-hand side of formula (10) in order to obtain the complete formula (7). Moreover, for methodological reasons, in order to bring the content of the mathematical equation as close as possible to the described reality, in situations involving the occurrence of a 'discrete time with measurable time intervals',[31] parameters α, β and γ (which are called elasticities[32] after the economically meaningful quantities are inserted) should be transformed into average intertemporal shares calculated according to the linear interpolation procedure. The use of more specific shares as elasticities, which is a more general category, is, as already mentioned, related to the assumptions of perfect competition and constant returns to scale in the economy; nevertheless, in the accounting practice this solution makes the determination of these elasticities possible. Some methodologies depart from these assumptions (e.g. the OECD methodology presented in Figure 1 does not include the assumption of constant returns to scale), but this necessitates completing the accounting with additional, empirically acquired data or estimating the parameters econometrically (from models), which is not always methodologically convincing.[33]

In result, the obtained equation is similar (though not identical) to Solow's original decomposition equation, where the used growth rates were written in the ordinary way and where no intermediate consumption was considered (additionally, Solow did not distinguish sectors, which is theoretically possible and is sometimes done in certain methodologies (e.g. Kotlewski, 2017a, 2019) presented in the further

[31] In some theoretical studies discrete time is not taken into account and its approximation is adopted in the form of continuous time. This allows the use of differential calculus – see e.g. Romer D. (2011).

[32] It is about the elasticities of response variable Y against explanatory variables X, K and L on the right-hand side of equation (7) in econometric terms.

[33] In the index decomposition method, i.e. also in KLEMS productivity accounting, empirical data is introduced into the calculus for the parameter related to the labour factor (nowadays usually denoted by the symbol α), and the missing values of the capital factor (β) parameter is calculated residually from formula $\beta = 1 - \alpha$. In the econometric method, on the other hand, both elasticities are estimated from a 'cloud'. The econometric method frees from assumptions of both perfect competition and constant returns to scale, but both elasticities are only certain approximations, whereas in the index method one of the elasticities (α) is based on high-quality empirical data. The econometric method is more often used in firm-level decomposition accounting. See for example: Ackerberg et al. (2015), Levinsohn and Petrin (2003), Olley and Pakes (1996). However, Diewert (1992) strongly advocates for the index method even in decomposition accounting at individual firm levels. In Poland, econometric methods have been addressed by Ciołek and Brodzicki (2016), Dańska-Borsiak (2011), Dańska-Borsiak and Laskowska (2012), Hagemejer (2006), Hagemejer and Kolasa (2011), Sulimierska (2014), and others. These methods do not meet the strict requirements of data processing set by the SNA (or ESA), but they allow insights into economic processes in the absence of relevant statistical data. They also enable analyses in additionally selected cross-sections. More recent econometric works of this type include e.g. Gradzewicz and Mućk (2019), Górajski and Błażej (2020), and Ulrichs and Gosińska (2020). A summary of firm-level decomposition methods can be found in Uguccioni (2016). The aggregate production function from microeconomic fundamentals was derived by Growiec (2006, 2008), Jones (2005) and others.

parts of this monograph[34]). The use of logarithmic expressions is of fundamental importance here as it facilitates the use of statistical methods. The relative increase in the value of A_{jt}^{Y} in percentage points, i.e. the contribution of MFP to the growth of gross output Y_{jt} in period t at the level of a given aggregation (sector also called industry) j, is in accounting practice calculated residually from equation (7), as the difference between the expression on the left-hand side connected with the value o Y_{jt} and the expressions relating to the values of X_{jt}, K_{jt} and L_{jt} on the right-hand side of equation (7), so that it remains always fulfilled. Thus, there is no need to determine the value of A_{jt}^{Y} to calculate its contribution in percentage points.[35]

An even more similar function to the one applied in Solow's decomposition is used in KLEMS productivity accounting for the GVA relative growth decomposition, since there is no contribution of intermediate consumption X_{jt}:

$$\Delta \ln V_{jt} = \bar{w}_{jt}^{K} \Delta \ln K_{jt} + \bar{w}_{jt}^{L} \Delta \ln L_{jt} + \Delta \ln A_{jt}^{V}, \tag{11}$$

where V_{jt} is the GVA generated in period t for a given aggregation (sector or industry) j and A_{jt}^{V} is the value-added-based MFP; the symbols have similar meanings as in equation (7), but except the capital services K_{jt} and the labour services L_{jt}, they have different values. It should be noted that analogous average shares $\bar{w}$ are not identical to average shares $\bar{v}$. Although they are given in percentages and calculated similarly to average shares $\bar{v}$, i.e. by linear interpolation, they are shares in GVA, not in the gross output.[36] They add up to unity, which implies not only the assumption of perfect competition but also the assumption of constant returns to scale (with respect to GVA, not to the gross output[37]). The MFP contribution calculated from the decomposition of the relative growth of GVA, i.e. $\Delta \ln A_{jt}^{V}$, is not identical in percentage points to the MFP contribution calculated from the decomposition of the relative growth of gross output $\Delta\, ln\, A_{jt}^{Y}$, since in the former case the baseline as reference is $V_{jt} = 100\%$, and in the latter case $Y_{jt} = 100\%$. These values should be connected by an appropriate ratio which, however, is disrupted by the variability connected with the possibility of the substitution phenomenon occurring between the

[34] As it is demonstrated in the further parts of this study, methodological simplifications are compensated by the possibility of performing some methodological deepening.

[35] The calculation of A_{jt}^{Y} is theoretically possible. It would be necessary to perform the calculation not for the increments but for the levels, which, however, may be subject to significant deviations from the actual values since the levels are usually less precisely known than their increments. See Kotlewski (2017b).

[36] Therefore: w_{jt}^{K} = (GVA – LR)/GVA and w_{jt}^{L} = LR/GVA, where LR is labour remuneration.

[37] Theoretically, there can be constant returns to scale at the level of GVA when the shares of labour remuneration and capital remuneration in GVA add up to unity, also in a situation where this is not the case with the shares in the gross output, as in the latter case there is an additional third variable – the share of intermediate consumption in the gross output – which can fluctuate independently. Theoretically, there may also be a situation where the three mentioned shares in the gross output add up to unity, but not the two above-mentioned shares in GVA.

basic production factors (labour and capital) and the intermediate consumption (and other deviations) mentioned before. This ratio is the quotient between the value of gross output and GVA Y_{jt}/V_{jt}, by which the MFP contribution obtained from the decomposition of the relative growth of gross output must be multiplied to obtain, in theory, a value corresponding directly to the MFP contribution obtained from the decomposition of the relative growth of GVA. The possible divergence, proving the existence of the above-mentioned substitution, provides information on important processes taking place in the economy (as discussed in Chapter 2).

1.2.2. Gross output and intermediate consumption aggregation

The relative growth of gross output in a given sector j in period t, i.e. the expression on the left-hand side of the decomposition equation (7) is defined as the sum of the contributions of the relative growths in the value of the gross output for individual products i of sector j:

$$\Delta \ln Y_{jt} = \sum_i \bar{v}_{ijt}^{Y} \Delta \ln Y_{ijt}, \tag{12}$$

where $\bar{v}$ with appropriate indices denotes the average intertemporal value shares of aggregates of the following products i of given sector j in gross output Y_{jt} of whole sector j between periods t and $t-1$ (these shares are calculated similarly to the previous ones by linear interpolation), and Y_{ijt} is the gross output of individual products i in sector j between the above-mentioned periods t and $t-1$. Equation (12) is representative of the Törnqvist procedure, which in turn is applicable to the whole KLEMS productivity accounting.[38]

Similarly, the relative growth of intermediate consumption in each sector can be defined by the equation:

$$\Delta \ln X_{jt} = \sum_i \bar{v}_{ijt}^{X} \Delta \ln X_{ijt}, \tag{13}$$

where $\bar{v}$ with appropriate indices denotes the average intertemporal value shares of intermediate consumption aggregates for the following products i of given sector j in the value of intermediate consumption X_{jt} of entire sector j between periods t and $t-1$, calculated in the same way as the shares in equation (12), and X_{ijt} is the intermediate consumption in the production of individual products i of sector j between periods t and $t-1$.

[38] This is the Törnqvist quantity index. On the other hand, in many American accounts the Fisher index is frequently used. The aggregation results for the two indices differ only slightly. See Dean et al. (1996) and Milana (2009).

According to the work on the EU KLEMS methodology done by Timmer et al. (2007a), most theoretical analyses indicate that purchase prices should be used when calculating the shares for intermediate consumption, hence these shares include indirect taxes such as VAT and trade and transport cost margins, but they do not include subsidies. For the capital and labour factors, the problem of choosing purchase prices does not arise. These operations are performed regularly in the framework of the existing SNA (more precisely ESA), and the data for KLEMS productivity accounting were made available by the National Accounts Department of Statistics Poland.

For many analyses, it is useful to divide the contribution of intermediate consumption into sub-contributions of its components (O'Mahony & Timmer, 2009, p. F374), which is one of the reasons why in the EU KLEMS methodology, intermediate consumption at the level of conventionally selected sectors j is initially aggregated into sub-contributions of enterprises' purchases of energy (E), materials (M) and services (S). These sub-contributions are added up to the total contribution of intermediate consumption in sector j, according to the equation:

$$\Delta \ln X_{jt} = \bar{w}_{jt}^{E} \Delta \ln X_{jt}^{E} + \bar{w}_{jt}^{M} \Delta \ln X_{jt}^{M} + \bar{w}_{jt}^{S} \Delta \ln X_{jt}^{S}, \tag{14}$$

where $\bar{w}$ with respective indices are the average value shares (calculated similarly to other shares, by linear interpolation) of energy, materials and services (denoted in superscripts as E, M and S) in the value of total intermediate consumption X_{jt} in sectors j between periods t and $t-1$. The symbols X_{jt}^{E}, X_{jt}^{M} and X_{jt}^{S} stand for the three aforementioned categories of intermediate consumption. For the sake of order, it is shown below that the sub-aggregation mentioned before is carried out according to the following equations:

$$\begin{aligned} \Delta \ln X_{jt}^{E} &= \textstyle\sum_i \bar{w}_{ijt}^{E} \Delta \ln X_{ijt}, \\ \Delta \ln X_{jt}^{M} &= \textstyle\sum_i \bar{w}_{ijt}^{M} \Delta \ln X_{ijt}, \\ \Delta \ln X_{jt}^{S} &= \textstyle\sum_i \bar{w}_{ijt}^{S} \Delta \ln X_{ijt}, \end{aligned} \tag{15}$$

where $\bar{w}$ with respective indices are the average value shares (calculated similarly to other shares, by linear interpolation) of energy, materials and services (denoted in superscript as E, M and S), consumed in the production of goods or services i in sectors j between periods t and $t-1$ in total intermediate consumption of the products or services X_{ijt}.

For the calculation of both gross output and intermediate consumption, the SUTs can be used as second best instead of the symmetric IOTs of Leontief (which unfortunately are only available periodically), as long as they are compatible with SNA and ESA and their updates. The tables for data transmission to Eurostat (TTs) are also compatible with ESA and the above-mentioned supply and use tables, and therefore also these can be used as a data source in KLEMS accounting. Where some data series are missing, appropriate estimations can be used (this issue will be further discussed in the subsequent part of the monograph).

1.2.3. Labour factor aggregation

The growth of labour factor services can be defined in a similar way to the growth of gross output and the growth of intermediate consumption, i.e. at the level of sectors j and in periods t. Very importantly, the term 'services' of the labour factor used here has a different meaning from services S, which are a component of the intermediate consumption, as already presented. This definition is made according to the equation:

$$\Delta \ln L_{jt} = \sum_l \bar{v}_{ljt} \Delta \ln H_{ljt}, \tag{16}$$

where L_{jt} is the value of labour factor services, l are the different types of labour factor, $\bar{v}$ with respective indices – average shares of the remuneration values of the different types of labour factor l in the remuneration of labour of whole sector j between periods t and $t-1$ (calculated similarly to the aforementioned shares as an arithmetic mean, i.e. by linear interpolation), and H_{ljt} is the number of hours worked in the given types of labour l in sectors j in periods t. It is assumed here that the 'services' of the labour factor of each type l expressed in value terms are proportional to the number of hours worked in each type of labour, and that workers in the given type of labour are paid according to their marginal productivity, which in turn is reflected in the share of the remuneration of the given type of labour $\bar{v}$ (with appropriate indices) in the total remuneration of all types of labour l of sector j between periods t and $t-1$. The EU KLEMS productivity accounting methodology distinguishes between these types of labour factor l according to the level of education (three levels), gender and age (in three groups), giving the total of 18 types of labour. The shares of these types of labour in sectors j are theoretically calculated in a similar manner to the shares of products i in sectors j in equations (12), (13) and (15). However, in equation (16), hours worked H_{ljt} may be remunerated differently depending on the type of labour l; as a result, the magnitude of H_{ljt} is not proportional to the corresponding magnitude $\bar{v}_{ljt}$, in contrast to equations (12), (13) and (15). Equation (16) therefore needs to be specified as:

$$\Delta \ln L_{jt} = \sum_{l=1}^{18} \bar{v}_{ljt} \Delta \ln H_{ljt}. \tag{17}$$

Equations (16) and (17) take into account both the effect of the heterogeneity of the labour factor from the point of view of its remuneration in $\bar{v}$ shares (with appropriate indices) and the physical growth of the labour factor (man-hours), expressed as the percentage increment of the logarithmic expression on the right-hand side. Therefore, equations (16) and (17) at the level of sector j determine the growth of 'labour services' in sectors j (which are unobservable in direct empirical examination) between periods t and $t-1$. At the level of sector j, this increment in labour services may be different from the physical increment in this factor, which is measured, for example, in hours worked (also in the number of workers, number of employed persons, number of full-time equivalents, or the number of hours paid). Generally, in KLEMS productivity accounting, this difference is referred to as an increase in the quality of work or, more commonly in the European variant (EU KLEMS), as a change in the 'labour composition'). The growth of labour quality can thus be defined according to the equation:

$$\Delta \ln LC_{jt} = \sum_{l} \bar{v}_{ljt} \Delta \ln H_{ljt} - \Delta \ln H_{jt}, \tag{18}$$

where LC_{jt} is labour quality or otherwise labour composition, and H_{jt} – the number of hours worked in sector j between periods t and $t-1$. Under the sign of the sum, there are symbols from equation (16), which can be specified as in equation (17). It follows from equation (18) that if the rate of increase in hours worked is greater (in relative terms) in those types of labour that are better remunerated, then the increase in labour quality – and hence the expression on the left-hand side of equation (18) – will be positive. Conversely, if the rate of increase in hours worked is lower (in relative terms) in the types of labour that are better remunerated, then the increase in labour quality – thus also the expression on the left-hand side of equation (18) – will be negative. However, this latter situation is rare.

With the concept of labour services, the contribution of TFP to the growth of a selected measure representing economic growth (relative growth of gross output, GDP or GVA) is reduced by the contribution of labour quality thus captured; this is related to the renaming of TFP into MFP, as discussed earlier. In addition to the sector-level component of labour quality, the literature also refers to the effect of labour reallocation between sectors (Stiroh, 2002). This gains additional importance when sectors have inherent, different wage levels, and there are shifts between them (these shifts are the above-mentioned labour reallocation between the sectors), which are not necessarily accompanied by real changes in labour quality. However, Stiroh's work on this subject still remains mostly theory only, as it has not been regularly implemented.[39]

1.2.4. Capital factor aggregation

By analogy to equations (12), (13), (15) and (16), it is possible to define the relative growth of capital-factor services (which, as in the case of labour factor services, are a different category from the services understood as a component of intermediate consumption). This is done by the following equation:

$$\Delta \ln K_{jt} = \textstyle\sum_k \bar{v}_{kjt} \, \Delta \ln R_{kjt}. \tag{19}$$

In this equation, the symbols L_{jt} and H_{ljt} used in formula (16) for the labour-factor services and hours worked were replaced by the symbols K_{jt} for the capital-factor services and R_{kjt} for individual types of fixed assets; here too, then, the Törnqvist procedure has been applied, as in the equations mentioned before. Therefore, average intertemporal value shares $\bar{v}$ of particular types of fixed assets k are calculated in the same way, i.e. by linear interpolation, as the arithmetic average over two periods, t and $t-1$, of the shares of their remuneration in the remuneration of the whole capital of sectors j (understood here as all fixed assets in sectors j).

In principle, the growth of the capital factor is defined within KLEMS[40] economic productivity growth accounting not as the increment in capital outlays into fixed assets, but as the increment in the value of capital services.[41] It is assumed that there is some identical nominal rate of return for all fixed assets within each sector (under constant returns to scale conditions), but different between the sectors (industries). Under such circumstances, and again assuming constant returns to scale, it is also possible to calculate the remuneration of capital in given sectors as the difference between GVA and the remuneration of the labour factor in those sectors,[42] which

[39] However, there are studies (e.g. Mankiw et al., 1992; Wang & Yao, 2003), in which a standard growth accounting with two types of capital, physical capital and human capital, is used, and the residual quantity is TFP.
[40] And in the OECD methodology as well.
[41] Theoretically, the arbitrage equation derived from the neoclassical investment theory, which was introduced by Jorgenson (1963) and Jorgenson and Griliches (1967), can be used for this purpose.
[42] This is discussed in more detail by Timmer et al. (2007a, pp. 33–34).

is standard practice in EU KLEMS productivity accounting, applied by almost all countries implementing this kind of productivity accounting (except Japan).

In equation (19), for each sector j, the relative increments in the value of different types of fixed assets are weighted by their shares in the remuneration of the total capital of sector j (in other words, in the remuneration of all fixed assets of sector j), so the relative growth in the capital services $\Delta \ln K_{jt}$ in sector j may theoretically have (and sometimes it actually has) a value significantly different than the relative growth of the sum of fixed assets (sum of capital) in that sector. If the stocks of better-remunerated fixed assets grow faster (in relative terms) than those of poorer-remunerated assets, then the relative growth of capital services is greater than the relative growth of total fixed assets in sector j. Conversely, if the stocks of better-remunerated fixed assets grow more slowly (in relative terms) than those of poorer-remunerated assets, then the capital services also grow more slowly than total fixed assets in sector j.[43] The procedure of aggregation by means of the Törnqvist quantity index makes it possible to determine the relative increase in the value of capital services, the use of which (alongside the category of labour services) in KLEMS productivity accounting distinguishes it from the classical Solow's decomposition. In the latter, instead of changes in the value of capital services, changes in the value of the stock of fixed assets, i.e., the value of sheer capital, were used. Thanks to the concept of capital services, the contribution of TFP to the growth of the chosen measure representing economic growth (gross output, GDP or GVA) is reduced by the difference between the contribution of capital services and the contribution of the stock of capital (with which the change of the name 'TFP' to MFP is associated). The Törnqvist procedure is used here to determine the contribution of capital services, because the 'capital services' variable is unobservable in direct examination, just as the 'labour services' variable is.

The applied procedure based on the Törnqvist quantity index is thus similar to the procedure aimed at determining the contribution of labour services, except that instead of the differentiation by type of labour l, a differentiation by type of capital k is used. This differentiation in the case of the labour factor is based on qualitative criteria related to sex, age and the level of education. The level of education is an element that can be shaped, which makes it possible to increase the difference between the rate of growth of labour services and the rate of growth of the supply of physical labour (preferably measured in hours worked). However, an oversupply of educated people can cancel this effect out, so it might not be clearly visible in developed countries. In contrast, what determines such differentiation in the case of the capital factor, is the breakdown of fixed assets into their different types, which

[43] The concept of capital value throughout KLEMS accounting here is understood as the value of fixed assets.

may vary in terms of quality. For example, the 2017 release of EU KLEMS distinguished the following types of fixed assets:

1. *computing equipment*,
2. *communication equipment*,
3. *computer software and databases*,
4. *transport equipment*,
5. o*ther machinery and equipment*,
6. *total non-residential investment*,
7. *residential structures*,
8. *cultivated assets*,
9. *research and development*,
10. *other IPP assets.*[44]

The above-mentioned shares are the value shares of particular types of fixed assets (which, for the sake of illustration, are presented above) in the remuneration of total fixed assets at the sector levels j. They are then different shares than the value shares for the relative growths of capital services for whole sectors j, as in equation (11), necessary to calculate their contributions to the relative growth of GVA at the sector levels; and they are also different than those in equation (7).

This division of fixed assets is slightly extended in comparison to the pre-2017 releases of EU KLEMS. However, in any case, the minimum breakdown of fixed assets into categories includes their three ICT types, i.e. computer equipment, telecommunication equipment and software (in the Polish conditions, these are not extracted from broader aggregates. The increments in these types of capital are combined using the Törnqvist quantity index, thus obtaining the growth of ICT capital services. The remaining types of capital are also aggregated using the Törnqvist quantity index, as a result of which the growth of non-ICT capital services is obtained. The equation presenting this distribution of capital-service inputs reads as follows:

$$\overline{w}_{jt}^{K}\Delta \ln K_{jt} = \overline{w}_{jt}^{KIT}\Delta \ln KIT_{jt} + \overline{w}_{jt}^{KNIT}\Delta \ln KNIT_{jt}, \tag{20}$$

where the superscript KIT indicates the values associated with the ICT capital, and the superscript $KNIT$ – the values associated with the non-ICT capital (the symbols are taken from Timmer et al., 2007a). Making the ICT capital category a separate sub-factor resulted from the belief that this capital has some special importance; however, this proved true only in the case of a few countries (e.g. for the USA in the 1990s).

Equation (20) is also an aggregation according to the Törnqvist quantity index, but only with two components. Therefore, it is possible to calculate the contribution

[44] IPP – intellectual property products.

of total capital services and the contribution of ICT capital services first, and then to calculate the contribution of non-ICT capital services residually, as the difference between the categories of total capital services and the categories of the ICT capital services. In such a case, the only difference between these methods are tool-related. Equation (20) is always fulfilled in accounting practice because one of its expressions is calculated residually.

The next procedure after determining the shares is inserting the relative growth rates in the value of capital services by sectors into equation (11) or into equation (7). In the EU KLEMS as well as in the OECD methodologies, only produced capital, without natural capital (in the form of e.g. land and natural resources) is taken into account, although it is controversial. However, for the time being it is not possible to proceed otherwise due to the lack of a developed and widely accepted common methodology.

1.2.5. Issues related to the labour factor

The fundamental problem in many cases of implementing theoretical solutions is related to the recording of the labour factor in inappropriate units, e.g. in persons instead of man-hours. What is also problematic here is the frequent use of man-hours paid rather than man-hours worked (and it is only the latter that is considered relevant in KLEMS productivity accounting). Another challenge is including self-employment in the calculations, as due to data availability constraints, various alternative estimation methods are used.

It should be noted that the measurement of the number of employees does not reflect changes in working time per employee, the fact that some people have more than one job, or the self-employed. The first step to improve the measurement of the labour factor is to adjust the number of employed persons to the number of working persons. To achieve this, the available method to estimate the volume of self-employment is adopted; this volume is assumed to be the difference between the categories of the number of working persons and the number of employed persons. In economic growth decomposition accounting, the category of full-time equivalent jobs is also sometimes used to compensate for the effects of part-time or multiple jobs. However, changes in the number of hours worked per full-time equivalents indicate that it is the number of these hours that is needed in order for the measurement of the labour factor to allow for an optimal estimation of the sizes of the services of this factor, according to the Törnqvist procedure. The above also justifies the necessity to calculate hours worked rather than hours paid, since, assuming that labour services are remunerated proportionally to the stream of hours paid at the level of particular types of labour, hours paid would hide some part of the information on labour quality and thus make its estimation less accurate.

If for a specific productivity account – and especially for KLEMS productivity accounting – statistical data in the form of hours worked for employed persons are available, it is best to use them adjusted by self-employment to an approximation of the number of hours worked by working persons. The use of hours worked for working persons calculated this way is common practice in KLEMS productivity accounting and is the basis for the calculation of labour services. However, in simplified approaches, it is acceptable to use categories other than hours worked in the productivity accounting.

The OECD productivity accounting (i.e., another kind of economic growth decomposition) is carried out for a larger group of countries and aims to ensure the comparability between them, so the labour factor is not decomposed there into different types of labour l (i.e. by age, sex and education level), as for a large part of these countries such data are not available. The labour factor is therefore decomposed into sectors only. However, in the OECD methodology, the breakdown by sector (industry) is more detailed.[45] In the case of the absence of available data on hours worked by sector, the aggregate number of hours worked for the whole economy is decomposed according to (in decreasing order of priority): full-time equivalent workers (adjusted number of working persons), full-time equivalent employees (adjusted number of employed persons), and the number of workers or the number of employed persons (similarly in Arnaud et al., 2011). As regards Poland, the OECD is currently using the third option, i.e. the number of workers. This method is also used by most of the EU KLEMS countries to disaggregate sectoral aggregates by age group, sex and education level (Timmer et al., 2007b).

In order to determine the volume of contributions of specific increments in labour services according to the aggregations selected in KLEMS productivity accounting in relation to the growth of the chosen measure representing the level of economic activity, i.e. the relative growth of GVA or the relative growth of gross output, these increments should be multiplied by appropriate weights. These weights

[45] The contribution of the change in the 'subtle structure' of wages (i.e., the changes in the share of particular wage categories) by type (quality) of labour (as in the EU KLEMS) and by sectoral breakdown (simplified to sections and groups of sections in the EU KLEMS, but more detailed in the OECD methodology) has a significant impact on reducing the size of Solow's residual. Thus, if labour composition (i.e. the scale and structure of wages) changes, the contribution of MFP might be much smaller than the classical Solow's residual, for which the contribution of wage change has not been calculated. When quality-adjusted measures of labour input are used in growth accounting instead of unadjusted hours worked, a larger share of output growth will be attributed to the factor 'labour' instead of the residual factor 'productivity growth' (OECD, 2001, p. 47). The more detailed sectoral breakdown used in the OECD methodology partly compensates for wage differences related to labour quality (high-wage and low-wage sectors), which partially addresses this problem (OECD, 2001, p. 48), but on the other hand generates negative effects related to the vertical integration of firms. Therefore, in the OECD methodological framework, the decomposition of gross output (i.e. disregarding intermediate consumption) is not performed. Another potential consequence of the adoption of the OECD methodology is the allocation of secondary activities to inappropriate sectors.

are the shares of labour remuneration values in GVA or in the value of gross output in the aggregations selected in KLEMS productivity accounting, which results from the aforementioned assumption of perfect competition and constant returns to scale. In determining these weights, the important principle to consider is that it is a cost borne by the producer. All wage supplements should be therefore taken into account when determining the shares of labour as the ratio of labour remuneration values to GVA, according to the selected aggregations.

Most important here, however, is the adjustment of the remuneration of employed persons with the remuneration of the self-employed (to obtain the remuneration of working persons. The result of subtracting the remuneration of employed persons from the joint remuneration of all factors, which is identified with GVA, is the ‘rough’ remuneration of capital, and which consists of ‘pure’ remuneration of capital (the sum of net operating surpluses) and ‘mixed income’, i.e. the income of entrepreneurs who are at the same time employees in their own firms, and their remuneration as self-employed persons cannot be formally distinguished from their profits resulting from their ownership of capital. This problem is solved by assuming that the self-employed pay themselves the same wages per hour worked as is attributed to employed persons at the level of a given KLEMS aggregation (in case of difficulties in determining hours worked for some aggregations, a different measure for the ‘labour’ factor may be chosen from among those mentioned above). After the labour remuneration is adjusted, the remaining part of mixed income is added to the net operating surplus, to obtain thus enlarged capital remuneration. In the literature (OECD, 2001, p. 45), one can also find the opposite procedure, in which it is assumed that the capital held by self-employed persons has the same rate of return as the capital constituting the net operating surplus. This makes it theoretically possible to calculate this part of the mixed income with which the net operating surplus must be adjusted in order to obtain the theoretically full remuneration of capital. The remaining part of the mixed income should then be added to the remuneration of employed persons. However, the latter procedure is rarely used, mainly because the data on the ‘capital’ factor are usually of a much lower quality than those on the ‘labour’ factor. It is usually more effective to adopt some of the theoretical postulates used in the national accounts (SNA and ESA) and calculate residually the corresponding values for capital by subtracting the corresponding values for the ‘labour’ factor.

Determining the level of labour remuneration according to the aggregations selected in KLEMS productivity accounting is also affected by expenditure on training for employees and other forms of remuneration that are not a part of salaries, such as securities or options, which have relatively recently become a part of some employee remuneration. The future will show whether an adjustment of the current method of making the calculations proves necessary.

1.2.6. Issues related to the capital factor

The flows of capital services are not directly observable, so data on fixed capital stocks are necessary as the basis for their calculation in productivity accounting, due to the assumption that at the level of the lowest aggregations adopted, these flows are proportional to them. At the level of these lowest aggregations, the relative (!) growths of capital services and of fixed capital stocks will therefore be identical by assumption, and so they must be properly prepared before they enter the basic growth decomposition accounting. Firstly, the known sources of data (and often the only ones that are published), are the stocks of fixed assets represented in the prices of the periods when the investments were incurred, i.e. in inventory prices, also called historical or mixed prices. These prices do not reflect the current value of capital, which in the case of capital markets' ideal functioning should represent the expected compound stream of all future revenues discounted with an appropriate interest rate (this is the 'net present value' (NPV) methodology of capital). Only such a value can be considered as proportional (to some extent) to the current stream of capital services, in other words, as their approximation under some given conditions.

Furthermore, the current-price values must be converted into constant-price values for specific accounting purposes so as to cancel out inflationary effects (of a monetary or market provenance) and pure fluctuation effects in the rapidly changing capital markets. One of the intermediate procedures used in deriving the current value of capital (before it is converted into its value at constant prices) is the depreciation of fixed capital. The purpose of such depreciation is to reflect both the phenomenon of the withdrawal of old assets from the production of goods and services and the phenomenon of the partial loss of their productivity over time, e.g. due to wear and tear. A mathematical depreciation technique is thus used to account for the phenomenon of capital depreciation.[46]

In order to obtain the contributions of capital services to the growth of the selected measures representing the level of economic activity (as e.g. in KLEMS productivity accounting, to the relative growth of the value of gross output or to the relative growth of GVA), one has to multiply the stocks of depreciated fixed capital at constant prices at the lowest aggregations selected in a given accounting by appropriate weights which represent the shares in the remuneration of capital (under the assumption of the perfect competition and constant returns to scale, which is plausible at macroeconomic levels). This demonstrates that in the case of capital-factor services, one should proceed similarly to the case of labour-factor services.

[46] Basic concepts and ideas can be found, e.g. in: Biørn et al. (1989), Blades (1998), Hulten (1990), Hulten and Wykoff (1996), Jorgenson (1996), OECD (2001, 2009) and Triplett (1996).

Hall and Jorgenson (1967; see also: OECD, 2001, pp. 69–70) made the task of determining the remuneration of capital considerably easier when they adopted the principle that the remuneration of capital is equal to the difference between GVA and the remuneration of labour at the level of sectors *j*. Another method involves the proportional distribution of the aggregate, residually calculated remuneration of capital, i.e. the difference between the aggregate GVA and the aggregate remuneration of labour, according to the particular shares of the fixed capital of sectors *j* in the entire fixed capital (i.e. the fixed capital at the aggregate level). In this latter method, the assumption of constant returns to scale is only applicable at the aggregate level, since with this method of disaggregation at the level of sectors *j*, the remuneration of capital and the remuneration of labour do not add up exactly to GVA. However, if the quality of the available input data and the results obtained is sufficiently high, these differences are of no major importance to the analysis.

The possible distribution of the aggregate residual remuneration of capital by sector *j* through its residual calculation at the level of sectors is not yet its disaggregation by type of fixed capital within each sector *j*, which is necessary to calculate the contributions of the capital-service category and also to separate the ICT capital as distinct from the non-ICT capital. Therefore, in some situations (and for some countries), it may nevertheless be necessary to use the structure of fixed assets to distribute this aggregate residual remuneration of capital. In Polish conditions (due to the excellent quality of data on fixed asset stocks), this method seems to be the best, but in the light of theory it is inferior to the 'mixed' method presented below.

This 'mixed' method consists in applying a distribution by sector *j* of the aggregate, residually calculated remuneration of capital according to the structure obtained in the form of the residually calculated remuneration values of capital at the level of sectors *j* (for mathematical-tool reasons, they do not add up to the aggregate value). Subsequently, the resulting sectoral values are distributed according to the intra-sectoral structure of types of fixed capital, which is done on the basis of the assumption of a homogenous intra-sectoral internal rate of return.[47] This procedure is, however, more complicated and its results in the trial calculations did not prove to be unequivocally better in comparison to the direct distribution of the aggregate remuneration of capital by means of the structures of fixed capital alone. Nevertheless, the theory favours this method (OECD, 2001, pp. 69–70).

In contrast, the OECD methodology currently applied in the productivity accounting assumes, in relation to capital, that real economies may have inconstant returns to scale and imperfect competition. It therefore does not use the option of calculating residually the shares of remuneration of capital in GVA in order to use

[47] If the internal rate of return is homogenous within a sector, then the remuneration of capital is proportional to fixed assets within the sector. This means that the residual remuneration of capital at the sector level can be broken down by type of fixed assets within the sector.

them as weights for calculating the contributions of capital services to relative GVA growth, as the KLEMS methodology does. In the OECD methodology, the elasticities used as weights for the contributions of the growths of labour and capital factors' services to GDP growth (rather than to GVA growth, as in the KLEMS methodology) are calculated at sector levels as shares in total costs incurred by the sectors (not as shares in the GVA).[48] These are labour and capital costs incurred by the sectors. These costs are not necessarily the same as the sectoral remuneration values of the factors of production calculated from the GVA, although it can be assumed that the absolute cost of labour in the OECD methodology is generally the same as the absolute remuneration of labour in the KLEMS methodology (despite the fact that the labour factor is decomposed differently in each of the two methodologies). Therefore, the difference between the OECD and KLEMS methodologies in the value of sectoral aggregates concerns the capital factor more.

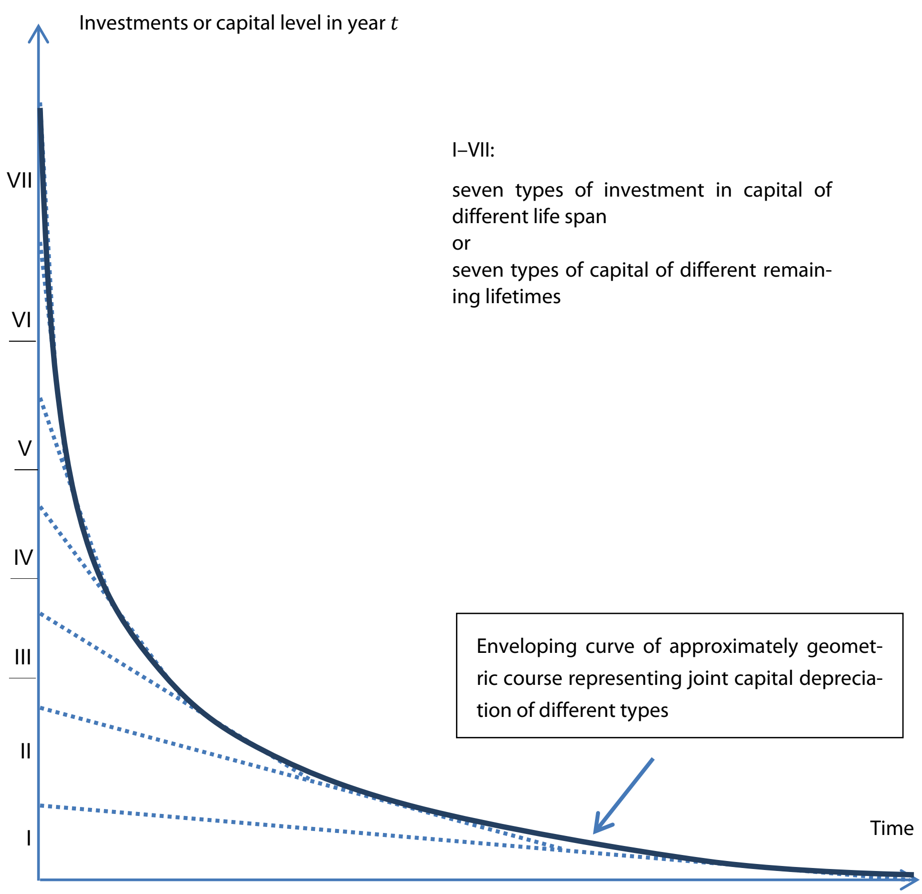
The basic way to determine the stock of fixed assets in the KLEMS (also the EU KLEMS) methodology is to take the value of a certain initial stock of fixed assets in a selected base year and to calculate changes in this stock by subtracting from it the value of the capital lost due to its depreciation, and by adding investments to it, according to the 'perpetual inventory' method. This is done in the national accounts, but taking into account different aggregations of NACE than in the EU KLEMS accounting. However, all the 34 EU KLEMS aggregations are either higher or equal to the NACE aggregations at division level, so the national accounts data can be used in KLEMS productivity accounting by aggregating them to the aggregations used in that type of accounting. The mathematical equations included in the EU KLEMS manual, derived on the basis of widely-accepted theories, allow capital depreciation to be calculated according to the given geometric capital depreciation rates in the USA, which can be used for all countries, and which are used by almost all EU KLEMS participants. However, due to the fact that some of these values are given in the manual as ranges of values, it is necessary to determine them for individual countries, which is usually done in the framework of national accounts.

The use of geometric capital depreciation raises doubts in a situation where data for individual fixed assets are only available in the form obtained as a result of the accountants' depreciation, e.g. linear depreciation. However, if one divides fixed assets into aggregates of capital with different lifetimes, the sectoral results are no longer negatively affected compared to the situation where fixed assets are depreciated geometrically. This mechanism is shown in Figure 2. As we can see there, the envelope curve for linear depreciations is approximately geometric, so similar to the envelope curve for geometric depreciations (not shown on the graph) for investments in all types of fixed assets accumulated in one graph (the working assumption here is that investments in different types of fixed assets have approximately equal

[48] Which is explained and justified, e.g. in the work of Wölfl and Hajkova (2007).

value). It is therefore enough to divide fixed assets into different types according to their life spans in order to determine the depreciation of capital. Division into a few types of fixed capital should be sufficient to ensure that the capital depreciation actually carried out for the aggregate fixed capital determined by different types of fixed capital after linear depreciation deviates only slightly from the ideal case of applying geometrical depreciation at the level of individual types of fixed capital. This will therefore be even more true in the case of depreciation aggregated from the level of individual capital plants in the economy. The envelope curve for geometrically modelled depreciations would look slightly different on a graph, but the selection of an appropriate depreciation rate on the basis of observation will ensure that the shift on the graph between the envelope for linear depreciations and the envelope for geometric depreciations is almost completely offset.

Figure 2. The mechanism of approximation to geometric capital depreciation



Source: author's work based on OECD (2009, p. 121).

Figure 2 also illustrates a different issue. If we accumulate capital with different remaining life spans in the graph (instead of investments in different types of fixed assets ordered according to the life spans of capital), also for the same type of fixed assets (with some modifications on the graph which do not undermine the fundamental assumption of geometric approximation), then we will obtain an approximately geometric, accumulated capital depreciation. It is therefore sufficient to select, on the basis of observations, the appropriate rate of geometric depreciation of capital to obtain results almost entirely equivalent to those of a potential depreciation that could be obtained by replacing a linear depreciation with a theoretically-sound geometrical depreciation, at the level of the lowest aggregations considered. This solution – based on the assumption that productive capital undergoes geometric depreciation – is working sufficiently well for most countries performing KLEMS productivity accounting that they adopt the above-mentioned American geometric depreciation rates (just with some minor modifications) in their calculations. Moreover, in this situation, in theory, the approximation with geometric rates on higher aggregations is obtained not only for linear depreciations, but also for other curves, including those of different shapes (provided that these differences in shapes are not too great, which generally can be assumed for an entity such as the economy).[49]

Problems with the international comparability of different types of capital may also arise from the use of different definitions of the IT (information technology) and CT (communication technology) sectors across countries. These definitions may be narrow and cover only the central components of devices with a specificity inherent in ICT technology, such as processors in the case of computers, but they may also be broad and cover numerous peripheral devices. Unfortunately, countries' practices differ widely in this respect, even within the EU KLEMS platform.

In addition, as regards the ICT capital, there are great discrepancies between countries in the way they calculate price deflation which occurs in the whole ICT products sector. Products in this industry are characterised by rapidly increasing quality and rapidly decreasing prices, both processes taking place simultaneously. The utility of ICT products for end users increases, which, however, is not accompanied by price increases. The latter fall as a consequence of rapid maturation and standardisation of products from this sector, which is accompanied by a fast-growing and continuous competition between the products that are already on the market, which leads to an imminent squeeze on profit margins. As a result of this situation, most countries on the EU KLEMS platform use negative inflation rates for the ICT sector. However, the differences are so large that they obscure interpretation

[49] For a detailed discussion on the course of depreciation, see OECD (2009, pp. 105–122). Depreciation with a geometric shape was first proposed by Matheson (1910). It was used on a regular basis by Jorgenson (1995a, 1995b, 1996). An extensive discussion on this topic can be found in Diewert (2005).

as no common methodological standard for the calculation of price inflation in the ICT products industry has yet been developed. This sector is distinct from the others, as in other industries, in general, different paces of improvement in the quality of products (understood as different pace of increase in their utility for the consumers) are reflected in the prices: changes in the degree of utility cause changes in the price relations between different categories of products.

This issue has not yet been definitively solved at the level of theory. Attempts have been made to use the hedonic price method (HPM) validation (see Baltas & Freeman, 2001; Cartwright, 1986, pp. 7–9; Cole et al., 1986, pp. 41–50; Triplett, 1987, pp. 630–634). However, this solution has not yet been universally accepted (Hulten, 2009, pp. 19–21). In a situation where types of data collected on the ICT industry vary greatly between countries, there are also great discrepancies between price indices adopted by individual countries for the ICT sector, even for countries with similar economies, such as those present in the EU KLEMS platform. The broadly-understood ICT market is not clearing fast enough for price elasticity to even these disparities, as is assumed in the neoclassical economic theory. Consumers become accustomed to ICT products of ever-higher quality, before the market can appreciate this quality in the form of price increases, as if ICT products were becoming obsolete before they had time to evolve from innovative to standard products.

Problems with international comparability also arise from the initial (base) condition of the stock of fixed assets, which in different countries can be estimated to a different extent, by different methods, and according to different initial years. However, the longer the time series, the less important the initial stock of fixed assets becomes, and so as time passes, the problem resolves itself. The correct assigning of capital services' rental (leasing) is also an issue. Capital services which are not owned by the users tend to be automatically treated as some type of intermediate consumption rather than a component of the capital factor (at the same time, countries differ considerably in their degree of vertical integration, so intermediate consumption is hidden to a larger or lesser degree in formalised or informal transactions within vertically integrated firms). Capital leasing is part of the already-mentioned phenomenon of the substitution of production factor inputs by intermediate consumption inputs, resulting in an overestimation of the MFP contribution to the relative GVA growth. In some countries which use KLEMS productivity accounting, attempts are made to remedy this problem, but so far they have lacked a developed standard for other countries to follow.

Upon the conclusion of the analysis of the contribution of capital services to the chosen output entities (gross output, GDP or the most often adopted in growth (or productivity) accounting – GVA), it should be mentioned that sometimes a difficulty arises, relating to the participation of public capital in infrastructure investment, which should be treated in the same way as private investment in KLEMS-type or similar productivity accounting, in the framework of economic growth decompositions. Meanwhile, there are large sunk capitals in the public sector. To allow analyses without the impact of the public sector, a special aggregation of the market economy is distinguished in KLEMS productivity accounting, which consists of the aggregate of the entire economy without some NACE sections: L –Real estate activities, O – Public administration and defence; compulsory social security, P – Education and Q – Healthcare and social assistance.

1.2.7. KLEMS productivity accounting system

The KLEMS productivity accounting system consists of specific standardised formulas representing algorithms already universally adopted by almost all implementers of this accounting. Basically, it is a system of combined accounts in formulas (7) or (11), i.e. accounts of relative growth of gross output (in percentages), of relative growth of GVA (in percentages) and of their components, i.e. accounts of the contribution of intermediate consumption (in percentage points), of the contribution of labour-factor services divided into two sub-contributions (in percentage points), and of the contribution of capital-factor services, also divided into two sub-contributions (in percentage points). A setup of five final variables[50] is used for the combined gross output growth decomposition accounting. These are:

$$\begin{aligned} GO\_\_Q &= \Delta \ln Y_{jt}, \\ GOconII &= \bar{v}_{jt}^{X} \Delta \ln X_{jt}, \\ GOconK &= \bar{v}_{jt}^{K} \Delta \ln K_{jt}, \\ GOconL &= \bar{v}_{jt}^{L} \Delta \ln L_{jt}, \\ GOconMFP &= \Delta \ln A_{jt}^{Y}. \end{aligned} \tag{21}$$

The symbols on the left-hand side of these equations are the codes used in standardised EU KLEMS, as well as WORLD KLEMS accounting for the final outcome variables.[51] They denote table matrices consisting of data for individual sectors j, spanned over time series for consecutive years t. *GO* denotes the gross output.

[50] These final variables are the calculated variables which have to be inserted into the basic formula (7).

[51] They have an established universal character, which is likely to be retained since it has been adopted by almost all countries performing KLEMS productivity accounting, so they are worth presenting.

The symbols on the right-hand side of equations (21) are the expressions – including those concerning factor contributions – from equation (7), calculated according to equations (12), (13), (16) and (17), preceded by parameters determined in a non-arbitrary way, which are the elasticities of the explained variable in relation to the explanatory variable which they precede. These elasticities, under appropriate assumptions adopted in economic theory, are average shares $\bar{v}$ (with appropriate indices, and calculated, like other average shares, as the arithmetic average over two periods) of factor remuneration values in the gross output. In other words, these are the weights to be applied to given factors at the appropriate level of aggregation. Factor increments multiplied by the weights are the contributions, i.e. the inputs to the relative growth of gross output. A weight lacking a symbol assumes the value of 1.

The contribution of the MFP growth in the last equation of equations (21) is calculated residually from basic-definition equation (7), therefore variable A (with appropriate indices) remains unspecified. In traditional KLEMS accounting, intermediate consumption variable X from the second equation of equations (21) ultimately has to be decomposed into three variables for the three subcategories of intermediate consumption: energy (E), materials (M) and services (S). These are:

$$\begin{aligned} GOconIIE &= \bar{v}_{jt}^{XE} \Delta \ln X_{jt}^{E}, \\ GOconIIM &= \bar{v}_{jt}^{XM} \Delta \ln X_{jt}^{M}, \\ GOconIIS &= \bar{v}_{jt}^{XS} \Delta \ln X_{jt}^{S}. \end{aligned} \tag{22}$$

As in equation (21), the symbols on the left-hand side are the standardised codes used on EU KLEMS as well as on World KLEMS platforms. Average shares $\bar{v}$ (also calculated as arithmetic average over two periods) with appropriate indices refer in this case only to the increments of the above-mentioned three components of intermediate consumption, and add up (in percentage points) to the total share of the intermediate consumption growth from the second equation of equations (21).[52] These three sub-factors indicated on the right-hand side of equations (22) by superscripts E, M and S should be inserted into equation (7) instead of the factor, i.e. intermediate consumption, indicated by superscript X. When the rates of change of the values under the symbol Δ are identical for all the three categories of intermediate consumption, this breakdown has no effect on the MFP contribution. If the

[52] According to Timmer et al. (2007a, p. 44). In contrast, in the work by O'Mahony and Timmer (2009, p. F377), it is assumed that these shares add up to unity and need to be multiplied by the share of total intermediate consumption.

pace of change is varied, then the pace of change of the whole factor is the weighted average of the pace of change of the three sub-factors, and likewise, it has no effect on the magnitude of the MFP contribution calculated residually. This procedure therefore serves only to decompose the intermediate consumption factor internally for the purpose of the economic analysis.

On the other hand, in the decomposition of the relative growth of GVA, the system of four final outcome variables which should be entered into the second basic definition equation (11) is used. These are:

$$\begin{aligned} VA\_\_Q &= \Delta \ln V_{jt}, \\ VAconK &= \overline{w}_{jt}^{K} \Delta \ln K_{jt}, \\ VAconL &= \overline{w}_{jt}^{L} \Delta \ln L_{jt}, \\ VAconMFP &= \Delta \ln A_{jt}^{V}. \end{aligned} \tag{23}$$

As in equations (21) and (22), the symbols on the left-hand side of these equations are the codes used in the EU KLEMS and in the World KLEMS, and the symbols on the right-hand side are the contributions to the relative GVA growth of the factors from equation (11). The symbol VA (value added) comes from the abbreviated English term GVA. Moreover, in the decomposition of the relative GVA growth, the capital factor growth variables divided into the growths of ICT-capital services and non-ICT-capital services are required, as are the labour factor growth variables divided into the growth of hours worked and the change in the composition of the labour factor representing the change in the quality of labour (along with variants described further in the chapter devoted to the implementation of KLEMS accounting in Poland):

$$\begin{aligned} VAconKIT &= \overline{w}_{jt}^{KIT} \Delta \ln KIT_{jt}, \\ VAconKNIT &= \overline{w}_{jt}^{KNIT} \Delta \ln KNIT_{jt}, \\ VAconH &= \overline{w}_{jt}^{L} \Delta \ln H_{jt}, \\ VAconLC &= \overline{w}_{jt}^{L} (\Delta \ln L_{jt} - \Delta \ln H_{jt}). \end{aligned} \tag{24}$$

In principle, the decomposition into subfactors also applies to the decomposition of the relative growth of gross output. The relevant equations for this decomposition take the following form:

$$
\begin{aligned}
GOconKIT &= \bar{v}_{jt}^{KIT}\Delta \ln KIT_{jt},\\
GOconKNIT &= \bar{v}_{jt}^{KNIT}\Delta \ln KNIT_{jt},\\
GOconH &= \bar{v}_{jt}^{L}\Delta \ln H_{jt},\\
GOconLC &= \bar{v}_{jt}^{L}(\Delta \ln L_{jt} - \Delta \ln H_{jt}).
\end{aligned}
\tag{25}
$$

Again, the symbols on the left-hand side of equations (24) and (25) are the codes used in the EU KLEMS and the World KLEMS. The capital previously denoted by K with the appropriate indices was separated into ICT capital denoted by KIT and non-ICT capital denoted by $KNIT$, both with appropriate indices under the symbol of natural logarithm *ln*. Their average shares, also calculated as the arithmetic average over two periods, denoted on the right-hand side of equations (24) and (25) by superscripts KIT and $KNIT$, understood as shares of their remuneration in GVA (for equations 24) or in gross output (for equations 25), are completely separable (as in the case of the three sub-factors of intermediate consumption). They are thus treated as if they were independent factors, and their shares add up (in percentage points) to the share of total capital K. Here, too, if the pace of change of the values under the sign Δ is the same for KIT and $KNIT$, this separation has no effect on the MFP increment calculated residually. However, if the pace of change varies, this separation may affect the calculated MFP contribution. This is because the remuneration of capital determining the weights does not reflect the stock of fixed assets whose value is under Δ, nor is it proportional to it. In practice, the following equations are often used:

$$
\begin{aligned}
VAconKNIT &= VAconK - VAconKIT,\\
GOconKNIT &= GOconK - GOconKIT
\end{aligned}
\tag{26}
$$

to eliminate tool deviations from the accounting at higher aggregations, assuming they are negligible for the analysis.

The contribution of the labour factor is calculated in a slightly different manner. In the last equation of equations (24), the contribution of hours worked (i.e. the contribution of the stock of labour) is subtracted from the contribution of the services of the entire labour factor in order to determine the contribution of the change to the labour composition (i.e. the labour quality), and the same common average share of the entire labour factor for both labour composition and hours worked (the last two equations) is used. This means that the labour factor is treated as a single factor internally separated into sub-factors, which, however, are not independent of each other. This separation therefore affects the contribution of the MFP growth calculated residually, since the pace of change of the better-paid work may be different from that of the poorer-paid work. Thus, while the contribution of capital-factor services has been split into the contribution of ICT-capital services and the contribution of non-ICT-capital services, the contribution of labour-factor services has been split into the contribution of the stock (or resource) of factor (i.e. hours worked) and the contribution of the difference between the contribution of labour-factor services and the contribution of labour-factor stock (resource), this difference being referred to as labour quality.

In the older versions of productivity accounting which did not distinguish labour quality (i.e. the labour composition), it remained within TFP, which can now be considered an early variant of MFP. However, if labour quality is separated, the labour factor is additionally supplemented (compared to the Solow decomposition) with the impact of the change in the share of better-paid labour against the share of poorer-paid labour. Moreover, thanks to the concept of capital services, the component related to the variability of the remuneration of capital was detached from TFP; nevertheless this variability is of lesser importance than in the case of the labour factor.

In order to ensure consistency in EU KLEMS productivity accounting, it has been proposed that the calculated figures for the decomposition of the relative growth of GVA also be used in the decomposition of the relative growth of gross output, according to the following equation (Timmer et al., 2007a, p. 16):

$$\Delta \ln Y_{jt} = \left(1 - \bar{v}_{it}^{V}\right)\Delta \ln X_{jt} + \bar{v}_{it}^{V}\Delta \ln V_{jt}. \tag{27}$$

Average share $\bar{v}$ (with appropriate indices), here also calculated as the arithmetic average over two periods, represents the average share of GVA in the value of gross output. What occurs here is $\bar{v}_{jt}^{X} = 1 - \bar{v}_{jt}^{V}$, so the consistency of the whole accounting is ensured by the functional link between the values calculated for basic-definition equation (11) and the values for basic-definition equation (7). The following functions between the variables for the decomposition of the relative growth of gross output and the variables for the decomposition of the relative growth of GVA are thus abiding:

$$
\begin{aligned}
GOconK &= \bar{v}_{it}^{V} VAconK, \\
GOconL &= \bar{v}_{it}^{V} VAconL, \\
GOconMFP &= \bar{v}_{it}^{V} VAconMFP.
\end{aligned}
\tag{28}
$$

The capital variable from equations (28) can be divided into an ICT-capital-related variable and a non-ICT-capital-related variable:

$$
\begin{aligned}
GOconKIT &= \bar{v}_{it}^{V} VAconKIT, \\
GOconKNIT &= \bar{v}_{it}^{V} VAconKNIT.
\end{aligned}
\tag{29}
$$

The same can be done with the variable for the labour factor:

$$
\begin{aligned}
GOconH &= \bar{v}_{it}^{V} VAconH, \\
GOconLC &= \bar{v}_{it}^{V} VAconLC.
\end{aligned}
\tag{30}
$$

The relation between the gross-output-based MFP growth and the gross-value-added-based MFP growth from equation (28) will take the following form:

$$
\Delta \ln A_{it}^{Y} = \bar{v}_{it}^{V} \Delta \ln A_{it}^{V} \tag{31}
$$

if we drop the codes used on the internet KLEMS accounting platforms. According to the reasoning presented in subsection 1.1, this relation is true if it is assumed that there is no substitution phenomenon between factor inputs and intermediate consumption inputs. The calculation of MFP contributions in the framework of both the decomposition of the relative gross output growth and the decomposition of the relative GVA growth can yield additional analytical conclusions (as discussed further in subsection 2.3).

KLEMS accounting faces the problem of a varying degree of vertical integration of firms in different countries, and therefore for many countries, the decomposition of the growth of gross output, in particular on the EU KLEMS platform, has been abandoned altogether, thus limiting their accounting to the decomposition of the relative growth of GVA. This is due to the fact that it is always possible to calculate the approximate value of the contribution of the MFP increment to the relative increment in the global gross output by means of equation (31) (recommended in the EU KLEMS methodology), providing that the above-mentioned substitution does not occur substantively.

The decomposition of the relative growth of GVA, due to the fact that it carries the most important information about the state of the economy of all the indicators within KLEMS productivity accounting, seems to be of fundamental importance to this kind of accounting. It constitutes its core, around which other decompositions are being developed. This other accounting activity is of a supplementary character, nevertheless it provides additional information which is both interesting and important to the analysis of the processes occurring in the economy.

Chapter 2
Implementation and development of KLEMS productivity accounting for the Polish economy

In the implementation of KLEMS productivity accounting, the basic assumption was to perform as first the decomposition of the relative growth of GVA only, because it is the carrier of the most important information about the economy, namely the resulting level of the economic activity. This measure also works best for international comparisons (using GDP is more cumbersome for theoretical reasons explained in Chapter 1). It is the methodological core of the whole accounting, being at the same time the starting point of all analyses based on factor decomposition of economic growth. However, even launching this first stage involved the need to solve the problem of availability of appropriate statistical data in Poland.

In the process of the implementation of KLEMS productivity accounting, it turned out that it was possible to carry out a deeper decomposition of the contribution of the labour factor to economic growth, which creates some new analytical opportunities. In parallel, during the development of the accounting, the 'deflators' for intermediate consumption became available, which made it possible to carry out a decomposition of the relative growth of gross output. Thanks to the comparison between two multifactor productivities, namely the one obtained from the decomposition of the relative growth of GVA and the one obtained from the decomposition of the relative growth of gross output, further possibilities in the area of economic analysis have opened up.

2.1. Implementation of GVA relative growth decomposition

Different regions of the world have specific common features that distinguish them from the others. This is partly due to geographical proximity, often also thanks to cultural similarity, neighbourhood imitation, and the membership in the same international economic (or other) organisations of an integrative nature. In the case of Europe, and especially Western Europe or EU countries, these specific features also occur and enable statistical services to obtain data of a similar kind, extracted according to fairly coherent principles. For example, in the KLEMS productivity accounting for European countries it is possible to adopt a common classification division into economic sectors in the framework of the NACE Revision 2 system (from the point of view of KLEMS productivity accounting, it is a close equivalent of the Polish PKD 2007 classification system[53]) on a scale of almost all of Europe, or at least on the EU scale. Despite these similarities, however, some significant differences still occur between European countries in terms of the availability of data for the purpose of KLEMS productivity accounting, which also applies to data on the Polish economy compared to other European economies.

In the light of the above-mentioned circumstances, it seems justified to carry out the KLEMS economic productivity accounting for the Polish economy on the basis of assumptions as close as possible to those used by countries long present on the EU KLEMS consortium[54] internet platform, which encompasses a relatively large and representative number of European countries. These assumptions include performing only the decomposition of relative GVA growth, as the decomposition of relative gross output growth is feasible for individual countries only when the respective national statistical offices have 'deflators' for intermediate consumption. The latter decomposition is therefore much more demanding from the point of view of the availability of relevant statistical data. Of some importance here are also issues associated with the comparability of the results of the decomposition of the relative growth of gross output at the international level, which are a consequence of large differences in the vertical integration of firms. In these circumstances, the presence of a relatively large number of countries varied in this respect on the European continent makes it even more difficult to perform internationally comparable decompositions of the relative gross output growth.

In addition, GVA is an entity very close to the commonly used GDP, and therefore appeals to the imagination of users accustomed to GDP as the primary measure of the level of economic activity. The GDP and GVA differ from each other in such a way that the former takes into consideration net indirect taxes. The value of taxes on products is added to GVA and subsidies on products are subtracted to obtain GDP on the basis of the assumptions adopted in the SNA and its European equivalent ESA, with subsequent updates. According to Statistics Poland's definition (which conforms to SNA and ESA), GVA is the term for GDP at factor prices

[53] The differences between the internationally agreed NACE Revision 2 classification system and the national PKD 2007 system, as specified by international agreements, occur at the level of much lower aggregations than the ones used in KLEMS productivity accounting.

[54] In the 2016 release of EU KLEMS, the ten countries which had long been performing the decomposition of relative gross-value-added growth in the framework of KLEMS productivity accounting were the following: Austria, Belgium, Finland, France, Germany, Italy, the Netherlands, Spain, Sweden and the United Kingdom. In the 2017 release, this group was expanded by Denmark, the Czech Republic, Luxembourg, Latvia, Slovakia and Slovenia; in the case of some of them, time series are very short. As regards the remaining countries present on the EU KLEMS platform, the aforementioned decomposition, which is the most important element of KLEMS productivity accounting, was not performed.

(also referred to as producer or base prices), whereas GDP refers specifically to GDP at market prices. In the economic sense the two measures are therefore essentially identical, and the difference between them is the applied accounting technique. The decomposition of the relative growth of GVA in KLEMS productivity accounting is considered as basic, because it carries the most significant information of fundamental importance for the analysis of economic growth, being at the same time methodologically consistent with the theoretical foundations of this kind of accounting.

In the light of the arguments presented above, it seems justified to limit the KLEMS productivity accounting for the Polish economy, at least in the first approach, to the decomposition of the relative growth of GVA only. The decomposition of the relative gross output growth should be treated as a certain development that can be carried out later, in the course of further work on the accounting, when the appropriate additional statistical data become available. In fact, the decomposition of the relative gross output growth has been performed in the KLEMS productivity accounting for Poland, but in a limited scope, which will be elaborated on later in this work

However, even when implementing the GVA relative growth decomposition, highly detailed statistical data are necessary, which additionally have to be transformed into a form suitable to be inserted into the algorithms used in KLEMS productivity accounting. Although eventually it turned out that sufficient data in a raw form are available for KLEMS productivity accounting, the innovative methods presented below proved necessary for their appropriate transformation. A similar need occurred in the case of many European countries (and probably also some from outside Europe), which had to transform data in original, often innovative ways (Timmer et al., 2007b).

2.1.1. Calculations related to the labour factor

In order to carry out KLEMS productivity accounting for the Polish economy, what is necessary, among other things, are data in a breakdown into different types of labour, in a way enabling the calculation of the labour factor services. Where these data are missing, at least the 'structure' contained in other statistical data must be available. This makes it possible – thanks to the method of proportionality transfer from one body of data to another – to break down the available data into data grouped according to different types of labour. Data from the Z-12 sample survey are precisely this kind of structure that has been used for the breakdown of data on the Polish labour market.

As regards the labour factor, data provided by the Z-12 sample survey are available for the following even years: 2004, 2006, 2008, 2010, 2012, 2014, 2016 and 2018 (and possibly for the subsequent even years), so this is a sufficient range[55] to perform the KLEMS productivity accounting for the Polish economy from 2005 onwards (KLEMS productivity accounting analyses annual relative growths, therefore data for one year backwards are necessary). To obtain adequate data for odd years, they had to be estimated by linear interpolation.

For the labour factor, the data available for 2004 for KLEMS productivity accounting provided by the Z-12 survey concern the following: the number of persons employed contractually full-time, the average gross hourly remuneration in PLN (Polish zlotys) per hour actually worked within regular working hours and overtime, over the whole year, by persons employed full-time, and the number of hours actually worked by persons employed full-time. From 2006 onwards, these data refer to all contractually employed persons, not only to persons employed full-time. They serve as the aforementioned structure, which is proportionally extended to estimate other data covering the whole labour market, so the possible inaccuracy resulting from this change has become negligible (more information on handling KLEMS productivity accounting input data will be provided further in the work).

Data from the Z-12 sample survey for the years 2004–2007 necessary for the KLEMS productivity accounting for the Polish economy are available in the PKD 2004 classification system only (i.e. NACE Rev. 1.1 Polish equivalent). Since 2008, they have been available in the framework of the PKD 2007 classification system (i.e. NACE Rev. 2 Polish equivalent). In addition, data for 2008 were also made available in the framework of the PKD 2004 classification system (by the Labour Market Department of Statistics Poland[56]), specially for the purpose of KLEMS productivity accounting calculations. A simplified correspondence between the two abovementioned classification systems (used by many countries performing KLEMS productivity accounting), applied only to the labour factor[57] decomposed into 14 sections or groups of sections,[58] made it possible to avoid the problem of data inconsistency resulting from the slight differences between the two classification systems. However, it was possible only when the data in the PKD 2004 (NACE 1) classification system were used to calculate the increments ('deltas') between the years 2007

[55] Before 2004, surveys on the labour factor ware carried out in a non-systematic way, so for the KLEMS productivity accounting for the Polish economy there may be difficulties in extending the necessary time series backwards, unless one refers to the 2007 release of EU KLEMS, in which a KLEMS-type decomposition was performed for the years 1996–2004 (discussed further), by making appropriate, rather far-reaching assumptions when estimating the missing data (Timmer et al., 2007b, pp. 121–129).

[56] The name of this department changed during the works on KLEMS productivity accounting.

[57] In Poland it was also used locally in the processing of data on the capital factor, which is discussed in the further part of this work.

[58] Some of the sectoral differentiation was abandoned in favour of a differentiation according to 18 types of labour by sex, three age groups and three levels of education attainment. Such specification is also adopted on the EU KLEMS platform.

and 2008, and the data in the PKD 2007 (NACE 2) classification system to calculate the increments between the years 2008 and 2009. In practice, this meant using data for 2008 available either in the framework of one or the other NACE classification, depending on the needs.

In the course of writing this monograph, it turned out that some data from 2006 onwards may become fully available in the PKD 2007 (NACE 2) classification system, and that the preparation of data for 2004 in this system might also become possible at some time in the future. This would most likely allow full consistency to be achieved in the next round of calculations. As a result, the use of the simplified correspondence in the form of the above-mentioned 14 aggregations would no longer be necessary. For the sake of order, this simplified correspondence between the PKD 2004 and PKD 2007 systems (NACE 1 and NACE 2 systems), as used in KLEMS productivity accounting, is presented in Table 2.

Table 2. Simplified correspondence between PKD (NACE) systems

PKD	Aggregations													
2004	A, B	C	D	E	F	G	H	I	J	K	L	M	N	O
2007	A	B	C	D, E	F	G	I	H, J	K	L, M, N	O	P	Q	R, S

Source: author's work based on information available on www.euklems.net.

The aforementioned data are, in accordance with the requirements of KLEMS productivity accounting, broken down by sex, three age groups (15–29, 30–49, 50+), and three levels of education (lower, secondary and higher, according to the standard criteria adopted in KLEMS productivity accounting[59]), i.e. in a division by 18 types of the labour factor that are created through all the possible combinations ($2 \times 3 \times 3$). Thus a matrix of data is created with an adopted 'horizontal' span encompassing 18 types of labour and a 'vertical' span encompassing 14 groups of sections and sections common for both of the PKD (NACE) classifications (the division into 'horizontal' and 'vertical' spans is conventional here). If all data were available in the currently-used PKD 2007 (NACE 2) classification system, which, as mentioned, may happen sometime in the future, then the 'vertical' span would encompass 34 groups of divisions and divisions of PKD 2007 (NACE 2), corresponding with the 34 lowest aggregations used in KLEMS productivity accounting.

However, the data from the aforementioned Z-12 sample survey concerns only ca. 7–8 million persons employed contractually, and thus do not account for the entire

[59] These criteria follow the generally accepted understanding of the division into lower, secondary and tertiary education, without going into country-specific variants.

labour market. Therefore, they can serve only as a structure which should be adjusted upward[60] by more comprehensive data concerning, at least in assumption, the entire labour market. Such data come from the BAEL survey (the Polish equivalent of the European survey on the labour force – Labour Force Survey), and have been made available for the purpose of KLEMS productivity accounting for the Polish economy in the form of TTs.

The vertically arranged structure of data from this source divided into groups of divisions and divisions compliant with the requirements for the form of TTs. divided into 64 aggregations of NACE 2 (further denoted by the A64[61] symbol, in accordance with the Eurostat convention), i.e. in this case a strict equivalent of the Polish PKD 2007, was converted into a vertical vector with the span of 14 sections and groups of sections (further, by analogy, denoted as A14). This vector was used to adjust upward 14 rows of the already-mentioned 18 × 14 matrix from the Z-12 survey (which for the years 2004–2008 is provided in the framework of the PKD 2004 classification system, and from 2008 onward in the framework of the PKD 2007 classification system, i.e. with one overlapping year) obtained after the aggregation of data from the Z-12 survey into A14 aggregations. This matrix was then expanded into an 18 × 34 matrix, i.e. by 34 aggregations (further denoted by the A34 symbol) in the PKD 2007 (NACE 2) system, required in KLEMS productivity accounting. If the data fully belonging to one classification system (PKD 2007 – NACE 2) were available, then the 'vertical' span of A14 aggregations could be replaced by a span of A34 aggregations, and the accounting operation mentioned above, i.e. the conversion of the resulting 18 × 14 matrix into the resulting 18 × 34 matrix, would become unnecessary. Data prepared in the way described earlier in this paragraph were used for further calculations, following the methodology described in Chapter 1. Figure 3 presents the procedure for preparing data for the accounting.

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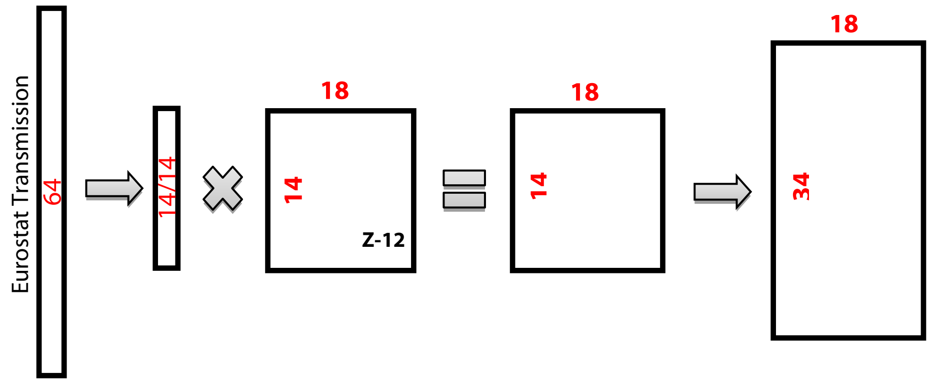
[60] A structure can be used to distribute the full 'true' data into smaller aggregations, or it can be adjusted upward with the full 'true' data to values closer to the real values. These two procedures produce mathematically identical results.

[61] There are 99 divisions in the NACE 2 classification system (actually NACE Revision 2), but some divisions in TTs are merged into common aggregations or are simply left blank. As a result, Eurostat requires a transmission of data broken down into 64 aggregations, which are traditionally denoted by the A64 symbol.

Figure 3. Preparation of data on the labour production factor



Source: author's work.

These procedures can also be presented formally, starting with the following equation:

$$H_{A14\epsilon TT} = \sum_{(A64\epsilon TT)\epsilon(A14\epsilon TT)} H_{A64\epsilon TT}. \tag{32}$$

In this equation, the $H_{A64\epsilon TT}$ values appearing on the right-hand side are the numbers of hours worked in the A64 aggregations into which the data in the TTs have been divided. The values for the A64 aggregations (at the level of NACE 2 divisions), mapped to the corresponding A14 aggregations (at the level of NACE 2 sections), need to be summed to obtain the $H_{A14\epsilon TT}$ values on the left-hand side of the equation, i.e. the number of hours worked in the A14 aggregations created for the data from the TTs corresponding to the selected division into 14 working KLEMS aggregations, introduced in order to enable the simultaneous use of the data available in the framework of the two NACE systems. This makes it possible to create a vertical vector consisting of the $H_{A14\epsilon TT}/H_{A14\epsilon(Z-12)}$ ratios, in which $H_{A14\epsilon(Z-12)}$ is the number of hours worked in the A14 aggregations created from the Z-12 sample survey data. This vector is used to adjust the $H_{A14\epsilon(Z-12),l}$ aggregations, i.e. the number of hours worked in the A14 aggregations created from the Z-12 sample survey data, but divided into 18 types of labour l:

$$H_{A14\epsilon KLEMS,l} = \frac{H_{A14\epsilon TT}}{H_{A14\epsilon(Z-12)}} H_{A14\epsilon(Z-12),l}. \tag{33}$$

Now what is left to be done is separating the A14 aggregations, whose values already comply with the KLEMS productivity accounting requirements, into the A34 aggregations used in this accounting:

$$H_{A34\epsilon KLEMS,l} = \frac{H_{(A34\epsilon TT)\epsilon(A14\epsilon TT)}}{H_{A14\epsilon TT}} H_{A14\epsilon KLEMS,l}. \tag{34}$$

In this equation, the A14 aggregations were separated, also by types of labour l, into the lower A34 aggregations used in KLEMS productivity accounting. For this purpose, a vector was used in equation (34), consisting of the ratios of the number of hours worked in the A34 aggregations (formed on the basis of data from TTs belonging to the corresponding A14 aggregations, also obtained from data from TTs), to the number of hours worked in the A14 aggregations.

If, in the future, the necessary data from the Z-12 survey were fully available only in the framework of the NACE 2 classification, then the use of aggregations A14 (common to the NACE 1 and NACE 2 systems) would be no longer necessary, and equation (32) would take the form:

$$H_{A34\epsilon TT} = \sum\nolimits_{(A64\epsilon TT)\epsilon(A34\epsilon TT)} H_{A64\epsilon TT}, \tag{35}$$

in which the A14 aggregations are replaced by the A34 aggregations. Consequently, equation (33) would then need to be replaced by:

$$H_{A34\epsilon KLEMS,18} = \frac{H_{A34\epsilon TT}}{H_{A34\epsilon(Z-12)}} H_{A34\epsilon(Z-12),18}, \tag{36}$$

and equation (34) would become superfluous.

According to this method, the labour factor has been calculated taking into account self-employment for both the number of hours worked and for the remuneration of this factor (compensations), at the level of each sectoral aggregation required in KLEMS productivity accounting. Self-employment has been taken into account in such a way that the values that are used in the accounting have been multiplied by the corresponding ratios of hours worked by working persons to hours worked by employed persons. This is common practice, one of the three standard methods for adjusting the values related with the labour factor to values including the self-employed (International Labour Organization [ILO], 2014, p. 173). Appropriately prepared data were inserted into equations (16) and (17).

Due to the fact that in equations (16) and (17) the Törnqvist procedure is used for aggregation, the contributions of the labour factor to the relative growth of GVA are the contributions of the labour factor 'services', not the 'resources' of the labour factor. During the calculations it turned out that the difference between the contributions of the labour factor services and the contributions of its resources are much greater in the Polish economy than in the economies of developed Western countries, for which KLEMS productivity accounting is also performed.[62] This difference is calculated and separated from the rest as the contributions of the 'labour composition' (labour quality) by means of equation (18). The quality of labour understood as the composition of labour (due to the specific way of calculating it demonstrated in the equations above) has certain consequences for further work on the KLEMS productivity accounting for the Polish economy carried out in Statistics Poland. Due to the relatively high importance of this factor to the Polish economy compared to other economies, research has been undertaken aimed at deepening the decomposition of the contribution of the labour factor (this subject will be elaborated on in the subsequent parts of the monograph).

As early as during the initial work on the KLEMS productivity accounting for the Polish economy, an alternative understanding of the labour factor found its way to the accounting practice, namely the scope of this factor was expanded by the effect of the growth of wages. The equation for the relative growth of labour quality understood in this way, denoted by the SC (*soft composition*) symbol, has the form of

$$\Delta \ln SC_{jt} = \sum_{l=1}^{18} \bar{v}_{ljt} \Delta \ln W_{ljt} - \sum_{l=1}^{18} \bar{v}_{ljt} \Delta \ln H_{ljt}, \qquad (37)$$

where W is the remuneration (compensation) of type of labour l in a given sector (industry) j in period t, which should be consistent with

$$\Delta \ln SC_{jt} = \Delta \ln W_{jt} - \Delta \ln L_{jt}, \qquad (38)$$

based on equation (16) and assuming that the first expression on the right-hand side of equation (38) is simply the sum of weighted wage increments. This is because, among other reasons, weights $\bar{v}_{ljt}$ for both expressions on the right-hand side of equation (37) are identical.

Referring to the current theory (ECFIN, 2014, pp. 9–10), labour composition (LC) or, in other words, labour quality, can be associated with labour efficiency (which is, in turn, associated with relatively long-term real changes), and the SC measure with the relative degree of labour usage (associated to a larger extent with the business cycle). In this way, the Solow residual understood as the TFP contribution, can be reduced[63] to the MFP contribution not only by the sub-contribution

[62] KLEMS productivity accounting (within the EU KLEMS consortium) is performed for a large group of developed Western countries, so it is relatively representative for them. It can also be assumed that in the case of countries other than Poland, but likewise undergoing the transformation from a centrally-planned economy to a market economy, this difference is also significant.

[63] Theoretically, if some contributions turned out to be negative, it could also be increased.

of labour composition, i.e. by the sub-contribution of its efficiency,[64] but also further, by the sub-contribution of labour usage, i.e. a value which (as shown above) can be directly associated with the SC value. Theoretically this means that the sub-contribution of capital usage would still be included in the contribution of such a reduced MFP to the relative growth of GVA.[65]

It is worth noting that in the light of some studies (Acemoglu, 2003; Klump et al., 2004), including Polish studies (Kotlewski, 2019), the significance of the sub-factor related to capital quality is generally decreasing in many national economies in the world. In the opinion of the author of this book, this is because technical progress is rapidly becoming embodied in sheer capital, and more specifically, in its price. Capital markets, unlike labour markets, clear relatively quickly. When capital is more productive, it quickly becomes more expensive because markets price it according to the NPV (net present value) principle, i.e. at the present value of the future expected income stream (the income stream that has been discounted with the expected interest rate). If, theoretically, it was possible to separate the business cycle-related sub-contribution of the degree of capital usage from MFP, then the only residual component of MFP would be the technical and organisational progress not embodied in labour and capital, potentially with its considered variant in the form of the capability of NACE sectors (industries) to capture value, which was discussed in Chapter 1. At the present stage of the development of KLEMS-type decomposition accountings, however, these are still only open theoretical speculations.

As a result, this first stage of KLEMS productivity accounting for the Polish economy was performed in two ways determined by two different approaches to the labour factor. The importance of the sub-contribution of labour composition for the Polish economy, and some interesting observations related to a different understanding of labour quality, as defined by equation (37), led to further research on the contribution of this factor, consisting in its decomposition into a larger number of sub-contributions, which will be further discussed in this chapter.

2.1.2. Calculations related to the capital factor

In KLEMS productivity accounting, when decomposing the relative growth of GVA, the category of capital services is used as a measure representing the capital factor instead of the traditional measure, i.e. the capital stock (resource). For this purpose,

[64] And by a much less important difference between the contribution of capital services and the contribution of capital stock (resource), related to capital quality (capital efficiency), which will be elaborated on further in the book.

[65] And the substitution effect between the production factors and the intermediate consumption (intermediate inputs), which was discussed in Chapter 1.

in the KLEMS-type decomposition accounting performed by Statistics Poland, the following types of capital were distinguished:[66]

1. *dwellings;*
2. *other structures and buildings;*
3. *transport equipment;*
4. *other machinery and equipment;*
5. *computer equipment;*
6. *telecommunication equipment;*
7. *cultivated assets;*
8. *intangible assets;*
9. *computer software.*

These names are identical or similar to those given in Chapter 1. There has been some evolution, which involves splitting the *intangible assets* (8th point) into the two categories listed in Chapter 1: *9. research and development* expenditure and *10. other intellectual property products* assets. In the practical implementation of KLEMS-related accounting, the 'cultivated assets' and 'intangible assets' are combined into the 'other assets' category, therefore the number of distinguished types of capital is still eight.

Dwellings (residential capital) could be classified as productive capital if most of the residential assets in Poland were held by rental companies that paid tax on revenues from capital.[67] Another difficulty in including residential capital in productive capital arises from the heterogeneity of the Polish housing market in terms of property rights – full property rights (private ownership of property), limited property rights with the right to dwell (cooperative ownership of property) and council (Polish: *kwaterunkowe*) dwellings, as well as with other types of restrictions on full property rights. Only recently have such discrepancies started to be removed from the housing market, which should be liquid and open. Therefore, the variety in property prices in Poland is not only related to the functioning of the market economy itself. For this reason, the KLEMS productivity accounting for the Polish economy was performed in two versions – taking and not taking into account the residential capital – in a way enabling users of the resulting data to freely choose the required variant. This, coupled with the two ways of understanding labour quality (as explained in the previous sub-chapter), gives two dichotomies, which ultimately leads to four ways of calculating the decomposition of the relative GVA growth

[66] The terms used in the National Accounts are in some cases different from those used in KLEMS productivity accounting, presented above. For example: 'office machinery and computer equipment' and 'radio, television and communication equipment' would be inappropriate for KLEMS.

[67] Accordingly, the KLEMS productivity accounting methodology analyses the potential for augmenting productive capital services by adjusting them by the housing property capital services.

(Figure 4) in the framework of accounting constructed in this way. If residential capital is not included in the accounting, then only seven kinds of capital are used to calculate the capital services,[68] which in the light of the rationale presented further, seems sufficient.

One of the serious shortcomings regarding the accounting for the capital production factor from the point of view of KLEMS productivity accounting is that the Polish National Accounts do not recognise certain kinds (categories) of capital as separate. The 5th and 6th categories, i.e. *computer equipment* and *telecommunications equipment*, are not separated from the 4th category labelled *other machinery and equipment*, and *computer software*, the 9th category, is not separated from *intangible assets,* which is the 8th one. These three kinds of capital, not separate from the others in the Polish conditions, are aggregated in KLEMS productivity accounting into one super-category of ICT capital, and the remaining kinds of capital into one super-category of non-ICT capital, using the Törnqvist quantity index. Separating the ICT capital from other categories is also a methodologically justified necessity, from the point of view of the consistency with the calculations for the European countries present on the EU KLEMS platform, to which the Polish KLEMS productivity accounting results are most often compared.

This is significant despite the fact that in the light of the performed KLEMS productivity accounting, the importance of the ICT capital in the Polish economy is modest, and therefore its impact on the results of the performed calculations is relatively small. One of the reasons for this is the relatively small difference between the contribution of capital services and the contribution of the stock of capital to the relative GVA growth in the Polish economy (this can be deduced on the basis of data from Statistics Poland's website dedicated to KLEMS productivity accounting for the Polish economy[69]). It is possible that a narrow understanding of the ICT capital in the KLEMS accounting performed for the Polish economy (e.g. without peripheral devices) contributes to this situation, which, in turn, results from the accounting technique adopted for estimating this capital. It would not be possible to estimate a broadly defined ICT capital (in which there is a larger share of domestic production) in a way useful for KLEMS productivity accounting. However, as various countries present on the EU KLEMS platform differ significantly in terms of the degree to which they include fixed assets in the ICT capital, such an approach to the accounting performed for the Polish economy seems to be right; the ICT capital understood in a narrow way occurs also in the KLEMS productivity accounting performed for countries such as Italy or France.

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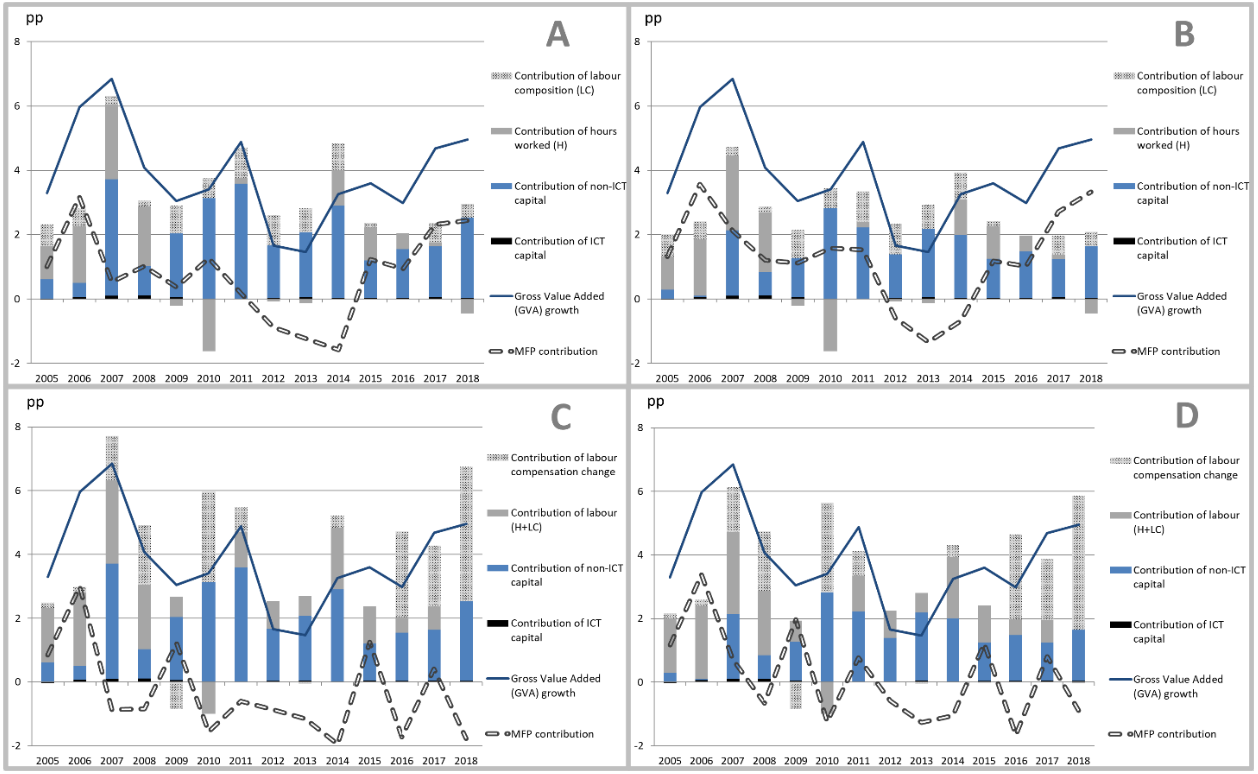
[68] The distinction between seven types of capital is also used in the decomposition accounts produced by the OECD (Figure 1).

[69] https://stat.gov.pl/en/experimental-statistics/klems-economic-productivity-accounts/.

Figure 4. Results of the relative GVA growth decomposition at the aggregate level



Note. Designations of decomposition variants by panels: A – capital without residential capital, labour quality understood as labour composition; B – capital with residential capital, labour quality understood as labour composition; C – capital without residential capital, labour quality understood as compensation level; D – capital with residential capital, labour quality understood as compensation level. For GVA, growth rates in percentage points are equal to growth rates in percentages.
Source: author's work based on data from https://stat.gov.pl/en/experimental-statistics/klems-economic-productivity-accounts/methodology-of-decomposition-in-klems-productivity-accounts-for-the-polish-economy,2,1.html, which are also available on the attached CD.

As regards the capital factor, the basic operation was therefore to separate these three types of ICT capital before aggregating them into a joint ICT capital category, using the Törnqvist quantity index. This was done on the basis of SUTs, and more specifically, on the basis of the figures in the investment outlays column for each of the three above-mentioned categories of ICT capital that were understood narrowly. These figures were then distributed with the use of a 'horizontal' (i.e. located horizontally in the SUTs) structure (vector) of software services by division, also taken from the SUTs. Earlier, this structure was transposed and aggregated into vertically stacked 34 lowest KLEMS aggregations (A34 aggregations), having assumed that within the sectoral aggregations, the software services are proportional to the three ICT capital categories (which is a viable approximation). Thus, it can be assumed that at the sectoral (industry) level, the volume of purchases of hardware and software is proportional to the demand for software services. This assumption can be extended to the telecommunications equipment due to the digitalisation of this equipment and its low importance compared to other categories of ICT capital.

It was further assumed that since ICT capital is ageing rapidly (it wears out and becomes obsolete at a fast rate), there is no need, given their very small value, to extract older parts of ICT capital from existing broader aggregates of capital. It was not considered necessary to include the pre-2005 ICT capital, the total value of which becomes much less than 10% of its initial value only after a few years, in order to determine the value of fixed assets in this category. Therefore, without taking into account the initial capital, the 2005–2010 outlays were summed up, after they had been depreciated for each year according to the US depreciation rates with a geometric trajectory, provided e.g. in the EU KLEMS manual (Timmer et al., 2007a). The ICT sector is characterised by price deflation that is highly varied across countries for which the decomposition accounting is performed on the EU KLEMS platform. Therefore it was decided to use the average deflation for ICT products from large European countries present on the EU KLEMS platform in the KLEMS productivity accounting for the Polish economy. The aim was to bring it closer to the European average (not precisely estimated yet), because price deflation in Poland most probably is not significantly different than in other European countries.

The 2010 stocks of fixed assets calculated in this way for the three categories of ICT capital were then extracted from the aggregates of the stocks of fixed assets in which they had previously been included. The stocks of fixed assets for the ICT capital for previous years were assumed to be in the same proportion to the stocks of other fixed assets (or, alternatively, the stocks of other fixed assets in the aggregates in which the ICT capital categories were included before their separation) as in

2010.[70] In the calculations for 2011–2018, the year 2010 was treated as a base year. Sometime later, SUTs until 2001 became available, which allowed these accounts to be reversed from 2005 to 2002 and the base year to be reversed from 2010 to 2007. This, however, did not substantially affect the calculation results. No further work to extend time series for the SUTs that would make it possible to move the base year further backwards is currently planned at the National Accounts.

Due to the fact that the SUTs were prepared on the basis of two separate classifications (PKD 2004 and PKD 2007, i.e. the equivalents of NACE 1 and 2), and no conversion of data from one classification to the other is planned at the moment, a simplified correspondence between these classifications by A14 aggregations used in KLEMS productivity accounting for the labour factor has been used where necessary. This ensures that any deviations for the labour factor resulting from this simplification should appear in the same places as the deviations for the capital factor, but work in the opposite direction. As a result, these factors add up better to the quantities associated with GVA at the sectoral (industry) level. The impact of this procedure on the residually calculated MFP contribution is therefore negligible.

KLEMS productivity accounting used the preceding years' SUTs in the calculations for the last two or three years. The compilation of such tables is a labour-intensive process, therefore their publication is delayed compared to other data published by statistical offices around the world (including Statistics Poland). The use of outdated SUTs is therefore a relatively common practice, as the structures, i.e. the relative ratios between the values in these tables, change very slowly.

As a consequence of this, the residually calculated aggregate remuneration of the capital factor – as the difference between the aggregate GVA and the aggregate remuneration of the labour factor – had to be de-aggregated (distributed) by the structure of the stocks of fixed capital, in order to enable the distribution of the remuneration of capital by eight types of fixed assets (including the three types of ICT capital) in each of the lowest KLEMS aggregations, i.e. in the A34 aggregations, and possibly also in the intermediate aggregations.

A slightly more complicated, alternative procedure can be used. It involves a 'vertical'[71] distribution of the remuneration of capital into A34 aggregations, by calculating them as the difference between GVA and the remuneration of labour at the level of the A34 aggregations, and its 'horizontal' distribution, by means of vectors spanning up to 7 categories (or 8, if the residential capital, i.e. dwellings, is taken into consideration) of fixed capital. These vectors are treated individually as structures for this proportional distribution from the 34 × 7 matrix of the stocks of fixed

[70] Otherwise, subsequent years before 2005 would have to be added as a starting point for calculations.

[71] 'Horizontal' and 'vertical' distributions are conventional terms that can be used interchangeably, depending on the technique adopted in the calculation work.

capital (where the number 34 stands for the A34 vertical aggregations, specified in KLEMS productivity accounting, along the lines of the NACE 2 divisions, i.e. PKD 2007, and 7 or 8 are the types of fixed assets used in KLEMS productivity accounting, broken down into three ICT capital types and four (or five with dwellings) types of non-ICT capital). This would imply a mixed-method approach for the de-aggregation (distribution) using two separate structures, one for 'vertical' distributions by A34 aggregation, and one for 'horizontal' distributions by seven or eight capital types. In this method, the remuneration of the capital and labour factors would balance exactly, not just approximately, with the GVA at the level of the distinguished A34 aggregations, as well as the intermediate aggregations used in KLEMS productivity accounting.

During trial calculations with this method it turned out that, due to its complexity, the deviations are not smaller than expected.[72] At this stage, this alternative data conversion operation was considered unnecessary and the first, slightly less complicated method was used. However, as described in Chapter 1, literature suggests (OECD, 2001, p. 69–70) that this more complex method is more relevant, although it is not universally accepted or applied in practice. Therefore, it is possible that at a later stage of the development of the accounting, when additional requirements arise (e.g. those related to the procedures of the implementation of the KLEMS regional productivity accounting for the Polish economy, described in Chapter 4), this method will be returned to. It should be mentioned here that the differences in the results between the two methods for the de-aggregation (distribution) of data on the capital factor are hardly noticeable at the level of higher aggregations, i.e. they have no consequences for the analyses. At the level of A34 aggregations, however, discrepancies may appear.[73]

Data on the capital factor initially prepared in this way were then used for further calculations, following the methodology described in Chapter 1.

There was a discussion about not separating the ICT capital (at a meeting of the Methodological Commission of Statistics Poland on 3 July 2015), which is the practice e.g. in Russia, and until recently also in Sweden and the United States of America[74]). However, because the procedure of separating the ICT capital is not laborious

[72] This method appears to be suppressing 'chimney' values, i.e. higher-yielding fixed assets, which is a substantial discrepancy.

[73] Therefore it might become necessary to revise the accounting for the capital factor, especially when carrying out KLEMS productivity accounting at a regional level (by Polish voivodships). Such a revision may prove necessary to maintain full methodological consistency in the whole accounting and compliance with theoretical assumptions.

[74] Data for this country on the EU KLEMS platform are provided with a separated ICT capital, but on the World KLEMS platform, which follows American rather than European methodological variants, this kind of capital is not separated. A similar, two-variant approach is adopted in Japan, a country present in some releases on the EU KLEMS platform. A certain American variant is applied in Canada – a country which, however, is not present on the EU KLEMS platform.

(thanks to the previously-developed apparatus of algorithms developed in the Excel programme), its calculations are continued for the Polish economy despite the minor importance of its sub-contribution to the relative growth of GVA, so potentially also to the relative growth of gross output. Nevertheless, the calculation of data without the separation of the ICT capital is also performed. The results obtained for the total capital (i.e. for the contribution of total capital services, which was calculated by splitting it only into four types of capital or five types – with residential capital) turned out to be almost indistinguishable from the results for the non-ICT capital (i.e. for the contribution of non-ICT capital services). It could be said that this was expected, given the small difference between the contribution of capital services and the contribution of the stock of capital at the aggregate level of the Polish economy. Larger differences of this kind (although also relatively small) can be observed only at the level of some NACE 2 sections (e.g. section J, related to the ICT industry, which is consistent with the expectations).

2.1.3. Other issues related to the implementation of basic accounts

The initially prepared data (calculated as above) were further processed as described in Chapter 1, except that for comparative purposes, four alternative conversion techniques were used as shown in Table 1, but only the results obtained through calculations using logarithmic expressions were published (according to the accepted practice). Due to two different understandings of labour quality and to whether residential capital was or was not taken into consideration, the KLEMS productivity accounting for the Polish economy was done in four versions, marked in Figure 4 with letters A, B, C and D, and on the Statistics Poland website devoted to KLEMS productivity accounting for the Polish economy with letters A, B, C and D, or A', B', C' and D' (the difference between these two series is explained below). Versions B and B' have the same assumptions concerning labour quality and residential capital as is the case in the countries for which the full decomposition of the relative GVA growth is performed on the EU KLEMS platform.

An additional dichotomy involves the possibility of performing decompositions both into the contributions to the relative growth of the aggregate GVA and into the contributions to the sectoral relative growth of GVA. The former were performed first, and are denoted by the letters A, B, C and D on Statistics Poland's website. The advantage of this approach is making the weights of the factor contributions at the level of the 34 KLEMS aggregations (A34 aggregations) and at the level of intermediate aggregations (already referred to above) visible in the aggregate relative growth of GVA.[75] Formally, equation (11) for these data should take the form:

[75] This approach makes it possible to construct the Harberger diagrams useful for the analysis (Timmer et al., 2011).

$$\overline{\left(\frac{V_{jt}}{V_t}\right)} \Delta \ln V_{jt} = \overline{\left(\frac{K_{jt}}{K_t}\right)} \overline{w}_{jt}^K \Delta \ln K_{jt} + \overline{\left(\frac{L_{jt}}{L_t}\right)} \overline{w}_{jt}^L \Delta \ln L_{jt} + \overline{\left(\frac{A_{jt}^V}{A_t^V}\right)} \Delta \ln A_{jt}^V \tag{39}$$

or

$$\overline{\left(\frac{V_{jt}}{V_t}\right)} \Delta \ln V_{jt} = \overline{\left(\frac{V_{jt}}{V_t}\right)} \overline{w}_{jt}^K \Delta \ln K_{jt} + \overline{\left(\frac{V_{jt}}{V_t}\right)} \overline{w}_{jt}^L \Delta \ln L_{jt} + \overline{\left(\frac{V_{jt}}{V_t}\right)} \Delta \ln A_{jt}^V, \tag{40}$$

with the resulting consequences for the following equations defining the components of equations (39) and (40) instead of the components of equation (11).[76] The fractional values are the average intertemporal shares of the sectoral values in the aggregate values for, respectively: GVA V, capital services K, labour services L and MFP A. However, the latter contributions, associated with the MFP customarily denoted by the symbol A with corresponding indices, are, according to the established practice, calculated residually – as differences between the remaining expressions of equations (39) and (40), thanks to which in practice these equations are always fulfilled.

Equation (39) produces the same result for the contribution of the residually calculated value associated with MFP as equation (40) if the growth rates of production factor services, i.e. labour services L and capital services K, are the same (these are the expressions under the 'delta' symbol). Otherwise, the above-mentioned equations produce discrepant results for this value. It is difficult, however, to interpret this discrepancy theoretically, as it means that when sectors that are more labour-intensive (i.e. with a greater sector-level share of labour factor services in the aggregate services of this factor) become even more labour-intensive, and capital-intensive sectors (i.e. with a larger share of capital factor services in aggregate services of this factor) become even more capital-intensive, the MFP contribution becomes smaller. The homogenisation between sectors, consisting in a convergent evolution in labour and capital intensity, would rather imply an increase in the contribution of this residually calculated MFP contribution. Therefore, equation (40) is preferred, which gives the results for MFP that are consistent with equation (11) and in many cases is convertible into the latter.

In the above-mentioned accounting method based on equation (11) and consisting in calculating the contributions to relative GVA growth at the level of sectoral (industry) aggregations A34 (the 34 KLEMS aggregations), the importance of the contributions in the aggregate of the whole economy is not apparent, but it is easier to compare the decompositions made at the level of separate aggregations from the point of view of their structure, since the same scale can be used for all the increments. To some extent, this is also possible when using equation (40) as a basis for calculating data for graphs that look the same but differ from graphs based on equation (11) in the vertical scale, which in the former case is varied.

[76] On both sides of equation (40), an identical coefficient appears in each expression. The equation has not been shortened, though, for the sake of the readability of the argument.

Table 3. Difference between values of increments in ESA 2010 system and values of increments in ESA'95 system ($\Delta \ln X_{ESA2010} - \Delta \ln X_{ESA'95}$) for the stocks of fixed assets

Aggregations by section and division of PKD 2007[a]	2005	2006	2007	2008	2009	2010	2011
Total	−0.00148	0.00158	0.00313	0.00165	0.00009	−0.00100	0.00997
Market economy[b]	0.00118	0.00155	−0.00016	0.00133	0.00071	0.00010	0.01635
A	0.00060	0.00064	0.00026	0.00072	0.00068	0.00053	0.00063
B	0.00028	0.00144	−0.00206	0.00043	−0.00078	−0.00055	0.00896
C	0.00069	0.00114	−0.00043	0.00138	0.00048	−0.00160	0.00751
10–12	0.00018	0.00041	0.00024	0.00099	0.00050	−0.00026	0.02459
13–15	0.00056	0.00192	0.00193	0.00229	0.00088	0.00132	0.00131
16–18	0.00053	0.00049	−0.00002	0.00001	0.00090	0.00000	−0.00001
19	−0.00082	−0.00106	−0.00061	−0.00003	0.00050	−0.00109	−0.00296
20–21	−0.00033	0.00136	0.00074	0.00266	−0.00153	−0.00173	0.00139
22–23	0.00061	0.00106	0.00016	0.00021	−0.00028	−0.00090	0.01844
24–25	0.00012	0.00036	−0.00029	−0.00007	−0.00016	−0.00018	0.00339
26–27	0.00320	0.00391	−0.02562	−0.00135	−0.00242	−0.00755	−0.00903
28	0.00773	0.00419	−0.00497	−0.00149	−0.00673	−0.00174	0.00168
29–30	0.00172	0.00296	−0.00113	0.00446	0.00372	−0.00490	0.00359
31–33	0.00089	0.00090	−0.00057	0.00053	0.00420	0.00084	0.00031
D–E	0.00001	0.00008	−0.00002	0.00036	0.00012	−0.00015	0.02062
F	0.00523	0.00216	−0.00545	−0.00136	0.00341	−0.00192	0.02754
G	0.00103	0.00228	−0.00151	−0.00125	0.00026	0.00004	0.02160
45	0.00209	0.00211	−0.00040	−0.00206	0.00164	−0.00275	−0.00149
46	0.00092	0.00250	−0.00239	−0.00234	−0.00027	0.00012	0.01781
47	0.00081	0.00191	−0.00085	−0.00029	0.00047	0.00034	0.03129
H	−0.00022	−0.00012	−0.00025	0.00006	0.00007	−0.00017	0.03387
49–52	−0.00024	−0.00012	−0.00026	0.00005	0.00005	−0.00020	0.03425
53	0.00086	0.00025	0.00034	0.00118	0.00146	0.00167	0.00143
I	0.00345	0.00153	−0.00160	−0.00092	0.00079	−0.00053	−0.00039
J	0.00399	0.00307	0.00196	0.00398	0.00216	0.00581	0.01324
58–60	0.00158	−0.00087	0.00437	0.00276	0.00777	−0.00124	0.00171
61	0.00167	0.00086	−0.00014	0.00153	0.00090	0.00485	0.01834
62–63	0.03674	0.04793	0.02097	0.00768	−0.02035	−0.01485	−0.05312
K	0.00329	0.00192	−0.00087	0.00238	0.00263	0.00371	0.00208
L	0.00007	0.00007	0.00002	0.00003	0.00002	−0.00003	−0.00621
M–N	0.02604	0.03298	0.01347	0.01078	0.00820	−0.00666	0.00863
O–U	−0.02471	0.00275	0.02706	0.00347	−0.00801	−0.00746	0.01462
O	−0.06739	0.00267	0.06833	0.01388	−0.02100	−0.02677	0.00299
P	0.00545	0.00512	0.00480	0.00067	0.00128	0.00420	0.02687
Q	0.00202	0.00050	−0.00023	−0.00177	−0.00092	0.00072	0.02381
R–S	−0.00420	0.01490	0.00167	0.01077	0.00614	0.01208	0.01432
R	0.00438	−0.02308	−0.00568	−0.01301	−0.00712	−0.01194	0.00226
S	0.00179	0.05860	0.02381	0.04484	0.03984	0.04126	0.04791

a Letters denote NACE 2 sections, numbers denote NACE 2 divisions. b Market economy, following the KLEMS definition, encompasses the whole economy except sections L, O, P and Q.

Note. White boxes – less than one-thousandth of a percent, grey boxes – thousandths of a percent, blue boxes – hundredths of a percent.

Source: author's work.

Tables made later, during the performance of the second method (based on equation (11)), were placed in the same sheets in Excel files as tables made for the first method, but under the latter, and labelled A', B', C' and D'. Most countries, including the EU KLEMS countries, perform decompositions on the basis of equation (11) only, so for the purpose of international comparisons carried out at the sectoral (industry) aggregation level, it is better to use option B' than option B. The disadvantage of this approach is that sectoral aggregations, even when properly weighted, do not always add up to the aggregate level of the economy (this problem lies in the calculation tool, so it applies to all countries performing KLEMS productivity accounting).

Switching from ESA'95 to ESA 2010 generated another complication, as not all data were converted from one system to the other and, even more importantly, some data are not planned to be converted at all (e.g. supply and use tables from before 2010). Because of that, it is sometimes necessary to use data from source tables made in the framework of different ESA systems. In order to check whether this is acceptable, an analysis of deviations was carried out by comparing the differences in the relative growth rates of fixed assets. The results of this analysis are presented in Table 3. As can be seen, the deviations that may result from a mixed use of data from both systems are negligible from the point of view of the needs of KLEMS economic productivity accounting (the largest differences occur around the transition from one statistical system to the other, i.e. in 2011, but they are still negligible).

2.1.4. Comparison of Statistics Poland's accounts with the 2007 EU KLEMS release

The current requirements for KLEMS productivity accounting preclude its performance for the Polish economy on the EU KLEMS platform, as its consortium[77] does not have data necessary to do it. However, the international collaboration under the leadership of the Groningen academic centre (Rijksuniversiteit Groningen) resulted in the extension of the 2007 release of EU KLEMS to the vast majority of European countries. Due to the large size of the platform, very loose data imputation rules and many simplifications, including some for the Polish economy, were adopted (Timmer et al., 2007b, pp. 121–129). Nevertheless, also thanks to the voluntary transmission of data by Statistics Poland to the creators of KLEMS productivity

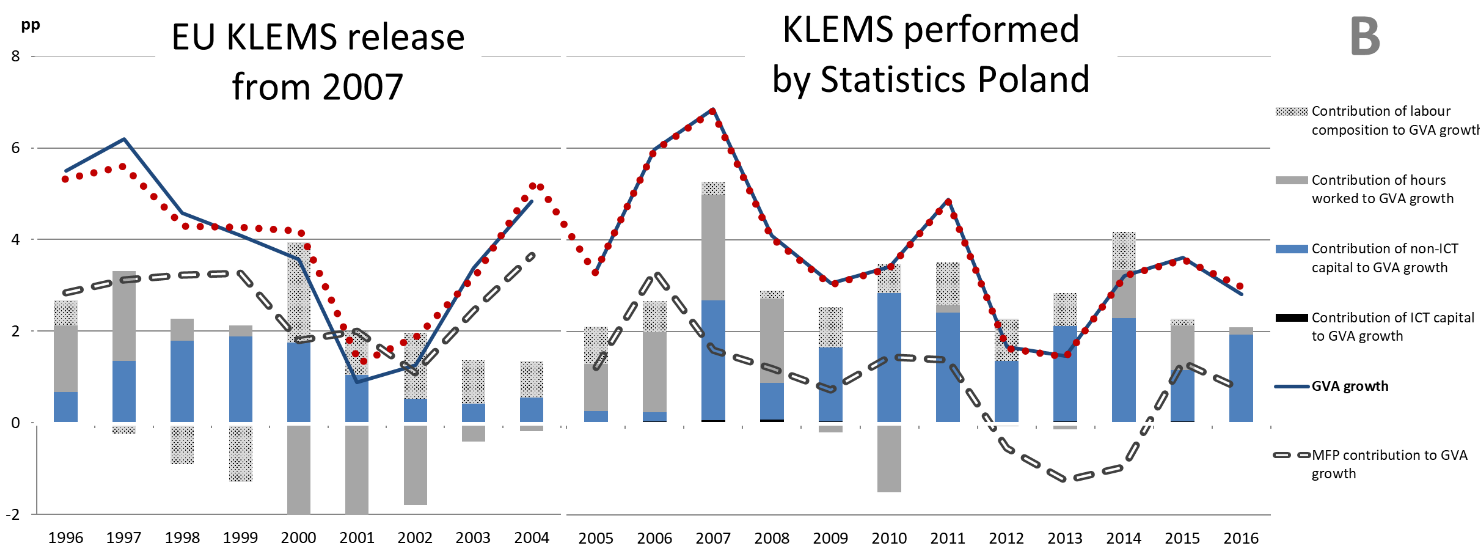
[77] The EU KLEMS platform is primarily the University of Groningen's (Rijksuniversiteit Groningen) online platform for publishing KLEMS productivity accounting data for a group of countries for which it is possible to obtain relevant statistics. The ambition of the creators of the EU KLEMS platform was to form a consortium under the leadership of the University of Groningen, consisting of representatives of European national statistical offices and academic centres. One of the results of this international collaboration was the 2007 EU KLEMS release, which also produced a basic version of the KLEMS productivity accounting for the Polish economy. However, after the 2007 EU KLEMS release the consortium did not continue in such a broad membership.

accounting from the University of Groningen, they were able to carry out a decomposition of the relative growth of GVA for the Polish economy for the years 1996–2004, and they even published data for the decomposition of the relative growth of gross output. This decomposition involves the extraction of the contribution of the labour composition, i.e. labour quality (which was done in the decomposition of relative GVA growth, but not in the decomposition of relative gross output growth), but the contribution of the capital services was not separated in this decomposition into sub-contributions of ICT capital and non-ICT capital. Capital services were calculated in the framework of this decomposition on the basis of fixed assets separated into five categories only.

Figure 5 compares two KLEMS-type decompositions of the relative GVA growth: one performed for 1996–2004 by the University of Groningen and the other performed for 2005–2016 by Statistics Poland. Due to the fact that the Department of National Accounts has revised the data for the period 1995–2003 (Główny Urząd Statystyczny [GUS], 2003), as well as numerous updates, revisions and verifications resulting from the international collaboration, including within Eurostat (Jeznach & Leszczyńska-Luberek, 2013) have been introduced, coupled by some degree of systemic inconsistency at the international level (Miguła et al., 2015), the accounting performed on the EU KLEMS platform should be considered outdated. This can be partially seen in Figure 5, where a dotted red line represents the relative growth of GVA for the entire 1996–2016 period, covering both the years when the KLEMS productivity accounting for the Polish economy was performed on the EU KLEMS platform, and the years when the KLEMS productivity accounting for the Polish economy was performed by Statistics Poland. It shows that this line overlaps with the continuous line representing the relative GVA growth only in the case of the latter. It is therefore clearly visible that the residually calculated MFP contribution would also have a slightly different course on the left graph for the EU KLEMS decomposition performed for the Polish economy.

However, the problem with the decomposition performed within the 2007 EU KLEMS release goes further. The calculation of contributions to the relative GVA growth uses elasticities that are (under the assumption of the perfect competition and constant returns to scale effects) the shares of remuneration values of the given production factors in GVA, whose levels and growth rates have changed. The heights of the bars illustrating the inputs of production factors on the left-hand side graph will also change, since these inputs are calculated (in percentage points) by multiplying the relative increments (in percentages) in the values assumed to represent production factors (i.e. the production factor services) by these elasticities.

Figure 5. Comparison of the KLEMS-type decomposition of the relative growth of GVA for the Polish economy from the 2007 EU KLEMS release with the decomposition performed by Statistics Poland (in version B)



Note. The variable marked with a dotted line represents the growth of GVA according to the current statistical methodology for the periods 1996–2004 and 2005–2016. For GVA, growth rates in percentage points are equal to growth rates in percentages.

Source: author's work based on data from www.euklems.net and https://stat.gov.pl/en/experimental-statistics/klems-economic-productivity-accounts/methodology-of-decomposition-in-klems-productivity-accounts-for-the-polish-economy,2,1.html (Statistics Poland's updated data on KLEMS are also available on the attached CD).

These revisions, of course, covered not only the GVA, but also production-factor data. As a result, from today's point of view, the picture on the left-hand side of Figure 5 appears as if seen 'through the eyes of a diver underwater without swimming goggles'. The only thing that can be said on its basis with a relatively high level of certainty is that in 2001–2002, the Polish economy experienced a sharp downturn, to which the negative contribution of hours worked was highly conductive, indicating the labour market crash in 2000–2002 (much deeper than the one in 2010 that happened after the 2007–2009 financial crisis). This downturn was also of a completely different nature than the economic growth downturn from 2012–2013, which was of a similar volume, but was caused to the largest extent by the negative contribution of MFP.[78]

The KLEMS-type decomposition carried out in the framework of the 2007 EU KLEMS release allows only very approximate observations. This is particularly important for analyses at the level of sectors (industries) adopted in KLEMS productivity accounting, i.e. the 34 lowest KLEMS aggregations (A34 aggregation according to the terminology used by Eurostat, i.e. divisions and groups of divisions of NACE 2), 13 higher KLEMS intermediate aggregations (groups of sections and sections of NACE 2) and for the second macroeconomic aggregate, i.e. the market economy, providing the differences observed for this latter aggregation were to be meaningful from the point of view of the aggregate for the whole economy. In the light of the above, sectoral analyses similar to those based on data from the KLEMS productivity accounting performed by Statistics Poland (Kotlewski & Błażej, 2020a) are not possible for the period for which result data for the Polish economy are available in the framework of the 2007 EU KLEMS release.

In contrast, the accountings carried out by Statistics Poland were revised and updated several times throughout their entire time series for the years 2005–2016, and this practice can be continued. They therefore correspond to the current methodological and analytical requirements imposed by the National Accounts system.

2.1.5. Sectoral specificity of KLEMS productivity accounting

Figure 4 shows only four decomposition variants at the aggregate level of the economy, but it should be underlined that the same is possible at the level of all 34 lowest KLEMS aggregations (A34 aggregations), 13 higher KLEMS intermediate aggregations, and the market economy.[79] As mentioned in the introduction, this monograph is not devoted to economic analyses, but to the methodology of KLEMS productivity

[78] An attempt to interpret this phenomenon analytically was made by Kotlewski and Błażej (2020b).

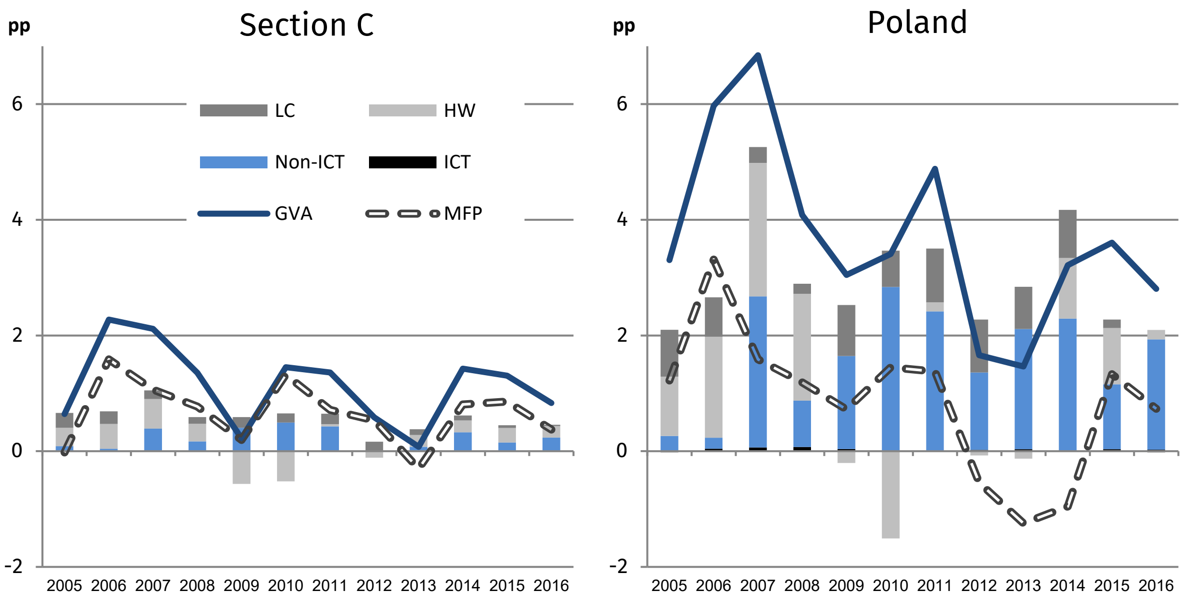
[79] The possibility of conducting analyses at the level of sectors (industries) of the Polish economy was shown by Kotlewski and Błażej (2020a).

accounting, therefore its analytical parts serve solely to demonstrate the importance of the performed accounts to economic analyses. It is worth pointing out that the NACE section with the largest contribution to the relative growth of GVA in the Polish economy is section C (manufacturing). This section plays the role of a trend settler in the economy. As the largest NACE 2 section in the economy, it constitutes its core, around which the remaining smaller sections revolve.

The downward trend in the relative growth of GVA in section C is less pronounced than in the whole Polish economy, as shown in Figure 6. This is the section that fuels economic growth, thanks to both its prominent role in the economy and a more optimistic trend. The special role of the manufacturing section is also confirmed by the analysis of the role of MFP, which is relatively more important (and has a larger share) in this section than in the whole economy. This phenomenon indicates that the Polish economy is undergoing the process of re-industrialisation.

KLEMS productivity accounting demonstrates that the contributions of particular production factors to the relative growth of GVA in NACE sections other than manufacturing are more diverse, which partly results from their natural specificity. In sections A (agriculture, forestry, hunting and fishing), G (wholesale and retail trade, repair of motor vehicles), K (financial and insurance activities), M–N (professional, scientific and technical activities, and administrative and support service activities), the sub-factors of the labour factor, i.e. the contribution of hours worked and the contribution of labour composition (labour quality) dominate, as these sections are labour-intensive by nature. On the other hand, in sections D–E (electricity, gas, steam, hot water and air-conditioning supply, and water supply; sewerage waste management and remediation activities), H (transport and storage) and L (real estate activities), the contribution of non-ICT capital dominates, as these sections are capital-intensive by nature (real estate activities are included here as the firms that deal with them often own capital facilities). The evolution of the contributions of individual production factors and their subfactors in these sections over time is also more varied than in the case of section C, which is the most similar to the whole Polish economy in this respect.

Figure 6. Results of the decomposition of the relative growth of GVA for manufacturing (section C in NACE 2) and for the whole Polish economy



Note. LC – labour composition contribution, HW – hours worked contribution, Non-ICT – non-ICT capital contribution, ICT – ICT capital contribution, MFP – MFP contribution, GVA – GVA contribution on the left-hand side graph or growth of aggregate GVA on the right-hand side graph. In order to assess the importance of the C section in the economy, the graphs were made in the same scale and values for the C section are contributions to values for the aggregate Polish economy. The right-hand side graph was made on the basis of data from version B of Figure 4.

Source: author's calculations based on data from https://stat.gov.pl/en/experimental-statistics/klems-economic-productivity-accounts/methodology-of-decomposition-in-klems-productivity-accounts-for-the-polish-economy,2,1.html and available also on the attached CD.

2.2. Developed decomposition of labour factor contribution

One of the key methodological developments of KLEMS productivity accounting for the Polish economy is that a deepening of the decomposition of the contribution of the labour factor to the relative GVA growth has become possible. The process involves splitting it into three or even four sub-contributions (instead of two, as practised so far). The fact that the decompositions within this accounting are performed at the sector level, i.e. at the 34 lowest KLEMS aggregations (A34 aggregations) and intermediate aggregations, creates new possibilities of analysing the business cycle and the labour market itself. As a result, not only is the basic macroeconomic analysis enhanced at the aggregate level, but an opportunity for finding links to other studies is also provided.

The standard decomposition of the contribution of the labour factor (understood as the contribution of labour services to the relative growth of GVA) into sub-contributions of labour quality and hours worked takes the following form:

$$\bar{w}_{jt}^{L}\Delta \ln L_{jt} = \bar{w}_{jt}^{L}\Delta \ \ln LC_{jt} + \bar{w}_{jt}^{L}\Delta \ln H_{jt}. \tag{41}$$

Equation (41)[80] is a transformation of equations (17) and (18), so the symbols are the same.

However, the analysis of the contribution of the labour factor to the relative growth of GVA can be deepened much more. The sub-contribution of hours worked from equation (41) can be decomposed into a sub-contribution of the number of working persons and a sub-contribution of hours worked per working person:

$$\bar{w}_{jt}^{L}\Delta \ln L_{jt} = \bar{w}_{jt}^{L}\Delta \ \ln LC_{jt} + \bar{w}_{jt}^{L}\Delta \ln M_{jt} + \bar{w}_{jt}^{L}\Delta \ln H_{Mjt}, \tag{42}$$

where:

$$\Delta \ln H_{Mjt} = \Delta \ln H_{jt} - \Delta \ln M_{jt}. \tag{43}$$

In equation (42), $\Delta \ln H_{Mjt}$ represents the relative growth in the number of hours worked H per working person, whose number is M, in sectors j between two discrete time periods – $(t-1)$ and t. In practice, this value is calculated residually (as in equation (43)), by subtracting the relative growth of the number of working persons $\Delta \ln M_{jt}$) from the relative growth of total number of hours worked $\Delta \ \ln \ H_{jt}$

[80] On both sides of equations (41) and (42), an identical multiplier appears at each expression. However, the equations have not been abbreviated to make the argument clear.

in sectors j between two discrete time periods – $(t-1)$ and t. The use of the technique of a residual calculation for the missing values guarantees that the equations above are always fulfilled in accounting practice. Hence this calculation method outperforms the one involving the division of the number of hours worked by the number of working persons at the level of given aggregations j, between time periods $(t-1)$ and t, and the observation of the changes in this ratio, as then no tool deviations occur.

In general, the reasoning behind this procedure is that the relative growth (and possibly a negative growth) in the number of hours worked at the level of the given aggregations (including the selected KLEMS aggregations) may be the result of two non-identical processes. One of them involves the possibility of a relative growth (fall) in the number of hours worked per working person and the other the possibility of a relative growth (fall) in the number of working persons. It is assumed here that these two types of increments (decreases) do not necessarily lead to the same consequences observed when analysing the functioning of the economy. In other words, this deepened decomposition of the contribution of the relative growth of the number of hours worked into sub-contributions of the relative growth of the number of working persons and the number of hours worked per working person may have a non-trivial and non-negligible significance when the KLEMS productivity accounting results are potentially used in economic policy-oriented analyses, e.g. for some variants of countercyclical policies.

When a negative shock occurs (related to, for example, an economic downturn or even a recession), the economy usually responds with a reduction in the hours worked in a selected period, usually a year. However, a situation in which the number of working persons is reduced, accompanied by a stabilisation (or even an increase) in the number of hours worked per working person is different from a situation in which economic adjustment takes the form of a reduction in the number of hours per working person accompanied by a small decrease in employment. In the former case, the social consequences are much more extensive and may turn into large declines in household consumption, with a contagion alike consumer spending reduction effect. In addition, the situation may entail high costs of restoring the previous level of employment, and thus consumption, due to the 'hysteresis effect'. In the second case, the social consequences of the shock are milder, with a much smaller fall in household consumption, thus its recovery is easier and faster, because households tend to maintain their consumption constant when their incomes decline only moderately. As a result, the course of a crisis, recession or economic downturn should be smoother when processes progress according to this model.

The development of an appropriate methodology which forms the basis of the analysis performed in the previous paragraph can make an important contribution to explaining the reasons for the different reactions of European (and non-European) economies during the great financial crisis of 2007–2009, as well as the reasons for the different paths of economic recovery. Preliminary studies performed for selected EU countries (using a simplified methodology) seem to confirm – in line with the present reasoning – the different reactions of European economies (GUS, 2014). These observations may prove even more interesting when a potential decomposition of economic growth by provinces or regions of individual countries is carried out.[81]

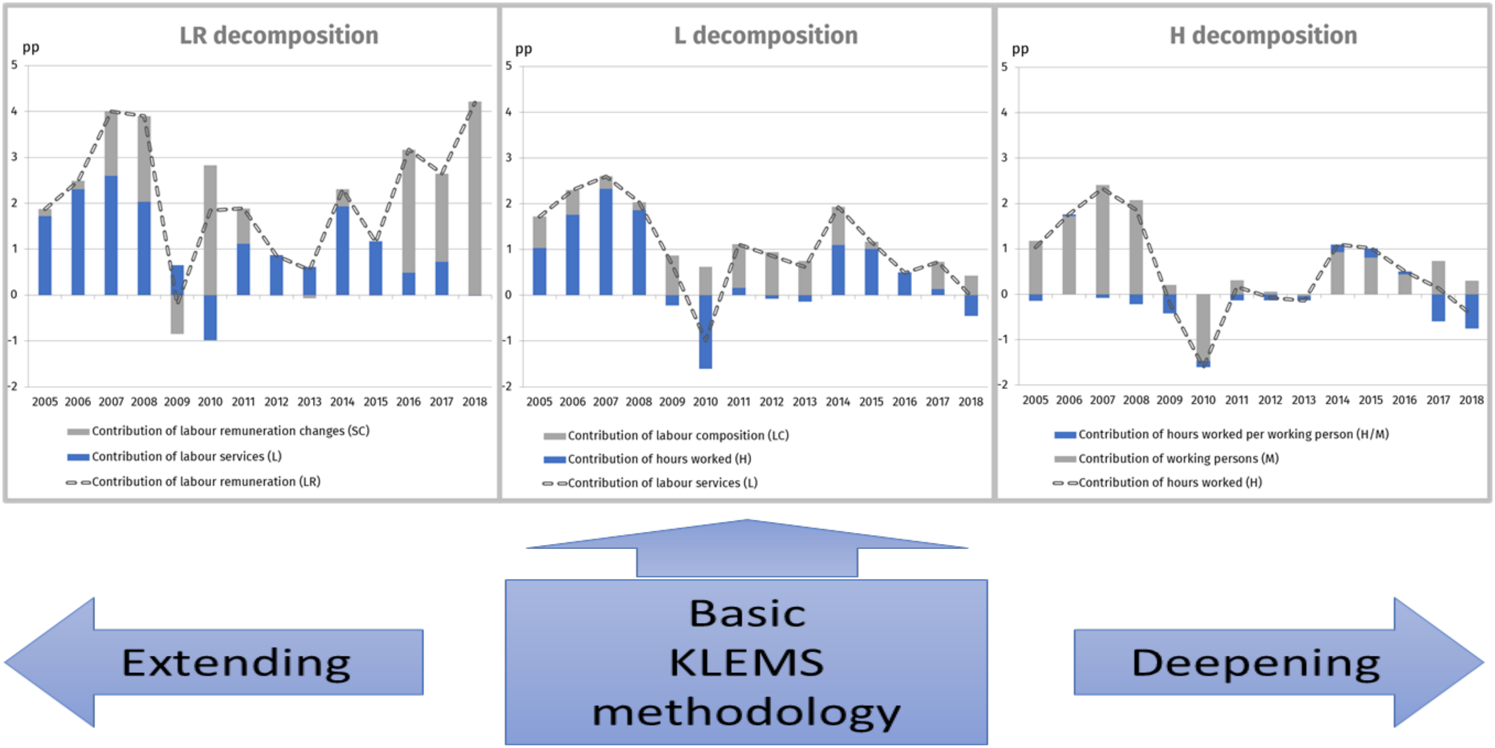
Figure 7 (the right-hand side graph) indicates that in 2009, the sub-contribution of hours worked per working person was negative. This is one of the reasons why the sub-contribution of the number of working persons remained positive, even when the contribution of total hours worked to the relative growth of GVA in the economy was negative. This seems to explain why the spending of Polish consumers has not declined as much as in the economies of other European countries (and not only). The argument above makes it easier to understand why the Polish economy avoided the recession in 2009, although other circumstances which influenced this situation should also be taken into account (e.g. a floating exchange rate, which contributed to the relative improvement in the balance of payments in crisis conditions, as well as a more traditionally functioning financial system).

In the economy there is a phenomenon of an overall remuneration growth, also in real terms. This growth may have a different relative rate in comparison with the growth of labour services L. Considering this difference in economic growth accounting would enable a full reflection of the phenomenon of growth of labour quality, understood as the effect of marginal productivity growth.

This is consistent with the reasoning of the supply-side theory of economics, according to which remuneration levels should equate with marginal productivity. From this it can be concluded that, ideally, the total effect of an increase in labour quality should include an additional dimension in the form of an increase in remuneration values as in equation (44).

[81] In Europe, the KLEMS regional productivity accounting has been done for Spain. Well-known non--European examples include China (see Kang & Peng, 2013), which has produced a growth decomposition accounting quite similar to KLEMS-type accountings.

Figure 7. Developed decomposition of the labour factor contribution



Source: author's work based on data from https://stat.gov.pl/en/experimental-statistics/klems-economic-productivity-accounts/methodology-of-decomposition-in-klems-productivity-accounts-for-the-polish-economy,2,1.html and available also on the attached CD.

The analysis of the contribution of the labour factor to the relative growth of GVA can not only be deepened as mentioned above but also extended. If the contribution of labour factor L, calculated as the contribution of labour services, is subtracted from the contribution of labour remuneration LR[82] (approximately from labour compensation), a sub-contribution of the relative change in the remuneration level SC[83] is obtained according to equation:

$$\bar{w}_{jt}^{L}\Delta \ln SC_{jt} = \bar{w}_{jt}^{L}\Delta \ln LR_{jt} - \bar{w}_{jt}^{L}\Delta \ln L_{jt}. \tag{44}$$

In this case (in line with this technique very often used in KLEMS economic productivity growth accounting), there is no need to determine the value of SC directly, since the value of the sub-contribution of pure relative remuneration change, i.e. the value of the left-hand side of equation (44),[84] can be calculated residually from the other expressions appearing in equation (44). It represents their respective difference on the right-hand side of the equation (since the value of labour remuneration is available within the National Accounts). In this way, the total theoretical contribution of the broadly-defined labour factor as the total remuneration of this factor, to the relative growth of GVA can be distributed over as many as four subcontributions. These can be summarised in a single equation as follows:

$$\begin{gathered}\bar{w}_{jt}^{L}\Delta \ln LR_{jt} = \bar{w}_{jt}^{L}\Delta \ln SC_{jt} + \bar{w}_{jt}^{L}\Delta \ln LC_{jt} + \\ \bar{w}_{jt}^{L}\Delta \ln M_{jt} + \bar{w}_{jt}^{L}\Delta \ln H_{Mjt}.\end{gathered} \tag{45}$$

In KLEMS productivity accounting, the contribution of LC is interpreted as the main manifestation of labour quality growth, i.e. labour efficiency growth, especially in a longer period, which is only partly reflected in the current level of remuneration values. The remaining contribution of the change in the SC remuneration level can be linked to the degree of labour usage, which is related with the business cycle, as well as to the reallocation effect between sectors, i.e. between A34 aggregations.[85] It is worth noting that the extending operation has already been carried out in principle at the stage of performing the basic accounting and is included in Figure 4 as variants C or D.

[82] The author's symbol, which has to be different from LC (reserved for the labour composition entity).
[83] The author's term, already used in the literature.
[84] On both sides of equations (44)–(46) an identical multiplier appears with each expression. However, the equations have not been abbreviated for the readability of the argument.
[85] This reallocation effect between sectors was investigated by Stiroh (2002). In the present work it is assumed that this effect is contained in the *SC* entity.

For clarity, equation (45) can be divided into three hierarchically stacked equations:

$$\bar{w}_{jt}^{L}\Delta\ln LR_{jt} = \bar{w}_{jt}^{L}\Delta\ln SC_{jt} + \bar{w}_{jt}^{L}\Delta\ln L_{jt},$$

$$\bar{w}_{jt}^{L}\Delta\ln L_{jt} = \bar{w}_{jt}^{L}\Delta\ln LC_{jt} + \bar{w}_{jt}^{L}\Delta\ln H_{jt}, \tag{46}$$

$$\bar{w}_{jt}^{L}\Delta\ln H_{jt} = \bar{w}_{jt}^{L}\Delta\ln M_{jt} + \bar{w}_{jt}^{L}\Delta\ln H_{Mjt}.$$

These three equations correspond to the three levels of the decomposition of the contribution of the labour factor shown in Figure 7. It can be seen that the fall in the sub-contribution of the relative change in the remuneration level, i.e. the fall of its contribution to the relative growth of GVA, shown on the left-hand side graph of Figure 7 (LR decomposition) probably delayed the onset of the collapse in the contribution of labour services observed in 2010 (visible on the middle graph – L decomposition). This was accompanied by 'labour hoarding' in 2009, shown on the right-hand-side graph (H decomposition) as a negative sub-contribution of hours worked per working person and a positive sub-contribution of the number of working persons.

The tables for this extension of the basic decomposition are denoted by the symbols E and E'. Those labelled E refer to contributions to the relative growth of the aggregate GVA, and those labelled E' refer to contributions to the relative growth of GVA at the sectoral (industry) level (A34 and intermediate aggregations). The expressions in equation (46) and earlier concerning tables E should be preceded by the $\overline{\left(\frac{V_{Jt}}{V_t}\right)}$ ratio, for accuracy, as in equation (40).

The additional and deeper decomposition of the labour factor is a specific development in the framework of KLEMS productivity accounting carried out for the Polish economy by Statistics Poland, a development that may also prove useful in analyses of the functioning of other countries' economies. It is possible to present this development also at the level of particular KLEMS sectoral aggregations, i.e. A34 and intermediate aggregations, for example to examine in which sectors (industries) the labour hoarding phenomenon occurred and what were its consequences.

2.3. Issue of gross output relative growth decomposition

The decomposition of the relative GVA growth into factor (labour and capital) services inputs and into residual MFP inputs, commonly practised in the framework of KLEMS productivity accounting, can be developed into a decomposition of the relative gross output growth, provided that 'deflators' are available for the additional component of this decomposition, i.e. for intermediate consumption (usually calculated as the ratio of values expressed in current prices to values expressed in constant prices, i.e. prices of the previous period, usually yearly). The non-performance of the decomposition of the relative growth of gross output in the framework of KLEMS productivity accounting, observed among many countries which already perform the decomposition of the relative growth of GVA, usually results from the unavailability of adequate data on intermediate consumption at constant prices that would enable the calculation of these deflators. Meanwhile, the National Accounts Department of Statistics Poland, during the paralell work on KLEMS productivity accounting, has developed appropriate statistics on intermediate consumption and gross output which enable the calculation of the above-mentioned deflators. Therefore, performing decomposition of the relative growth of gross output is possible.

When performing the relevant calculations, it is best to remain consistent with the calculations already carried out, related to the decomposition of the relative growth of GVA, i.e. to insert the values already calculated into the new equations. Thus, the starting point is equation (7), representing the decomposition of the relative growth of gross output at a given level of aggregations j between periods $t-1$ and t. This equation should be linked with equation (11) for the decomposition of the relative growth of GVA. Equation (11), on contributions to sectoral (industry) relative GVA growth, has its counterpart in equation (40), on contributions to the aggregate relative GVA growth. By analogy to equation (7), concerning the contributions to sectoral gross output relative growth, the corresponding equation concerning the contributions to aggregate gross output relative growth can be formulated. It will take the form of:[86]

$$\begin{gathered}\overline{\left(\frac{Y_{Jt}}{Y_t}\right)}\Delta\ln Y_{jt}=\overline{\left(\frac{Y_{Jt}}{Y_t}\right)}\bar{v}_{jt}^{X}\Delta\ln X_{jt}+\\+\overline{\left(\frac{Y_{Jt}}{Y_t}\right)}\bar{v}_{jt}^{K}\Delta\ln K_{jt}+\overline{\left(\frac{Y_{Jt}}{Y_t}\right)}\bar{v}_{jt}^{L}\Delta\ln L_{jt}+\overline{\left(\frac{Y_{Jt}}{Y_t}\right)}\bar{v}_{jt}^{X}\Delta\ln A_{jt}^{Y}.\end{gathered}\tag{47}$$

The accounting bond between equations (7) and (11) can be obtained if the relevant elements from equation (11) are inserted into equation (7), as follows:

$$\Delta\ln Y_{jt}=\bar{v}_{jt}^{X}\Delta\ln X_{jt}+\overline{\left(\frac{V_{Jt}}{Y_{Jt}}\right)}\bar{w}_{jt}^{K}\Delta\ln K_{jt}+\overline{\left(\frac{V_{Jt}}{Y_{Jt}}\right)}\bar{w}_{jt}^{L}\Delta\ln L_{jt}+\Delta\ln A_{jt}^{Y}.\tag{48}$$

[86] On both sides of equations (47) and (49)–(52) an identical multiplier appears at each expression. However, for the readability of the argument, the equations have not been shortened.

As indicated above, the elements of equation (11) related to the production factor (labour and capital) services need to be multiplied by the intertemporal average (between periods, usually annual, $t-1$ and t) ratios between GVA and the value of gross output at sectoral level j calculated by linear interpolation (similarly to shares v and w, with appropriate indices).

Equations (40) and (47) should be bonded in the same way as equations (7) and (11). This can be achieved by inserting the relevant components of equation (40) into equation (47):

$$\overline{\left(\frac{Y_{Jt}}{Y_t}\right)}\Delta\ln Y_{jt} = \overline{\left(\frac{Y_{Jt}}{Y_t}\right)}\bar{v}_{jt}^{X}\Delta\ln X_{jt} + \\ + \overline{\left(\frac{V_t}{Y_t}\right)}\,\overline{\left(\frac{V_{Jt}}{V_t}\right)}\bar{w}_{jt}^{K}\Delta\ln K_{jt} + \overline{\left(\frac{V_t}{Y_t}\right)}\,\overline{\left(\frac{V_{Jt}}{V_t}\right)}\bar{w}_{jt}^{L}\Delta\ln L_{jt} + \overline{\left(\frac{Y_{Jt}}{Y_t}\right)}\Delta\ln A_{jt}^{Y}. \tag{49}$$

As the formula above suggests, the elements of equation (40) relating to the production factor (labour and capital) services need to be multiplied by the ratios of GVA to the value of gross output at the aggregate level, calculated by linear interpolation (this is done in a similar way as for the v and w shares – with appropriate indices). Equation (49) can be shortened to

$$\overline{\left(\frac{Y_{Jt}}{Y_t}\right)}\Delta\ln Y_{jt} = \overline{\left(\frac{Y_{Jt}}{Y_t}\right)}\bar{v}_{jt}^{X}\Delta\ln X_{jt} + \\ + \overline{\left(\frac{V_{Jt}}{Y_t}\right)}\bar{w}_{jt}^{K}\Delta\ln K_{jt} + \overline{\left(\frac{V_{Jt}}{Y_t}\right)}\bar{w}_{jt}^{L}\Delta\ln L_{jt} + \overline{\left(\frac{Y_{Jt}}{Y_t}\right)}\Delta\ln A_{jt}^{Y}. \tag{50}$$

Equation (50) applies when all the expressions of this equation are the contributions to the relative growth of the aggregate gross output. Equation (48), on the other hand, applies when all the expressions are the contributions to the relative growth of gross output at the sectoral (industry) level, shown on the left-hand side of this equation. All ratios are calculated by linear interpolation as averages, over two periods (by analogy to the calculation of the shares).

The production factor services' inputs from equation (48) are disaggregated in KLEMS productivity accounting into the known factor sub-contributions associated with the ICT and the non-ICT capital and those associated to hours worked and labour quality:

$$\overline{\left(\frac{V_{Jt}}{Y_{Jt}}\right)}\bar{w}_{jt}^{K}\Delta\ln K_{jt} = \overline{\left(\frac{V_{Jt}}{Y_{Jt}}\right)}\bar{w}_{jt}^{KIT}\Delta\ln KIT_{jt} + \overline{\left(\frac{V_{Jt}}{Y_{Jt}}\right)}\bar{w}_{jt}^{KNIT}\Delta\ln KNIT_{jt}, \\ \overline{\left(\frac{V_{Jt}}{Y_{Jt}}\right)}\bar{w}_{jt}^{L}\Delta\ln L_{jt} = \overline{\left(\frac{V_{Jt}}{Y_{Jt}}\right)}\bar{w}_{jt}^{L}\Delta\ln H_{jt} + \overline{\left(\frac{V_{Jt}}{Y_{Jt}}\right)}\bar{w}_{jt}^{L}\Delta\ln LC_{jt}, \tag{51}$$

and the production factor services' inputs from equation (50) according to the following equations:

$$\overline{\left(\frac{V_{jt}}{Y_t}\right)}\,\overline{w}_{jt}^{K}\Delta\ln K_{jt} = \overline{\left(\frac{V_{jt}}{Y_t}\right)}\,\overline{w}_{jt}^{KIT}\Delta\ln KIT_{jt} + \overline{\left(\frac{V_{jt}}{Y_t}\right)}\,\overline{w}_{jt}^{KNIT}\Delta\ln KNIT_{jt},$$
$$\overline{\left(\frac{V_{jt}}{Y_t}\right)}\,\overline{w}_{jt}^{L}\Delta\ln L_{jt} = \overline{\left(\frac{V_{jt}}{Y_t}\right)}\,\overline{w}_{jt}^{L}\Delta\ln H_{jt} + \overline{\left(\frac{V_{jt}}{Y_t}\right)}\,\overline{w}_{jt}^{L}\Delta\ln LC_{jt}. \tag{52}$$

In equations (51) and (52), KIT stands for ICT capital, $KNIT$ for non-ICT capital, H for hours worked and LC for labour composition; however, there are other variants of an in-depth decomposition of the labour factor (more on this in the previous sub-section) that could be developed here.

For the contribution of MFP to the relative gross output growth to be comparable to the contribution of MFP to the relative GVA growth, it should be transformed according to the following equation:

$$\Delta\ln A_{jt}^{V*} = \overline{\left(\frac{Y_{jt}}{V_{jt}}\right)}\Delta\ln A_{jt}^{Y}. \tag{53}$$

This is because all expressions from equation (48) (after replacing the left-hand side of this equation with the corresponding GVA-related expression on the left-hand side of equation (11) and removing from the right-hand side of equation (48) the expression related to intermediate consumption X) should be multiplied by the inverse of coefficient $\left(\frac{V_{jt}}{Y_{jt}}\right)$ from equation (48) in order to obtain a corresponding equation for the decomposition of the relative growth of GVA closely related to equation (48) – its only difference from equation (11) is that it contains an asterisk.

In order to obtain a related equation for the decomposition of the relative GVA growth for equation (50), a similar operation has to be performed, involving the substitution of the left-hand side of this equation by the GVA-related expression from the left-hand side of equation (40). Furthermore, similarly to equation (48), the expression related to intermediate consumption should be removed from the right-hand side of equation (50), and the remaining expressions of this equation should be multiplied by the inverse of coefficient $\overline{\left(\frac{V_{jt}}{Y_t}\right)}$ from equation (50). Since in equation (50), in the expression related to variable A (concerning MFP), a certain coefficient is already present, which together with the above-mentioned inverse of coefficient $\overline{\left(\frac{V_{jt}}{Y_t}\right)}$ is shortened, equation (53) also applies to equation (50), as well as to equation (48). The above-mentioned coefficients have been written in a way showing their components for the clarity of the argument, but the last coefficient in equation (53) can also be written referring to the symbol from equation (31):

$$\overline{\left(\frac{Y_{Jt}}{V_{Jt}}\right)} = \frac{1}{\bar{v}_{jt}^{V}}. \tag{54}$$

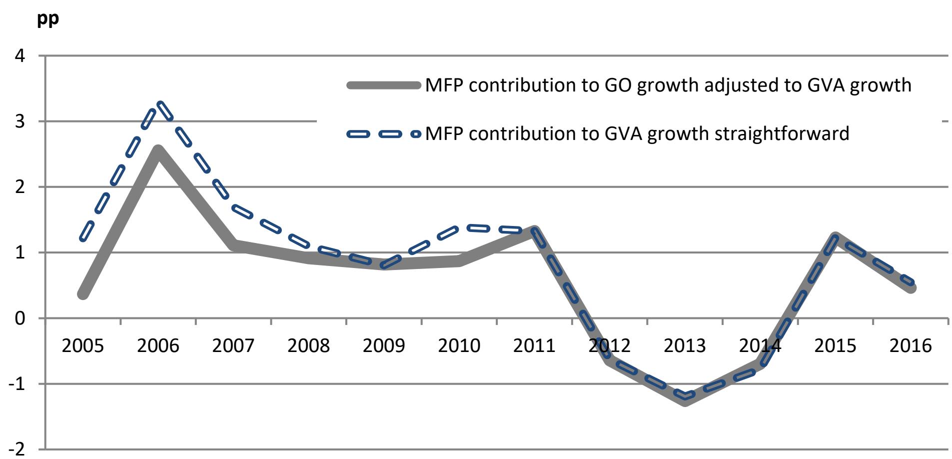
Having in mind the possible tool deviations, the result obtained should approximately fulfil the following condition:

$$\Delta \ln A_{jt}^{V*} \approx \Delta \ln A_{jt}^{V}. \tag{55}$$

This means that the result obtained by converting the MFP contribution to the relative gross output growth into the MFP contribution to the relative GVA growth should be in principle identical to the result obtained for the MFP contribution by the direct decomposition of the relative GVA growth. If the above is not the case, it means that the substitution between the production factors and the intermediate consumption is substantial, i.e. substantial changes in the economy are taking place mainly in the field of outsourcing and in the way some inputs are alternatively accounted as capital inputs or as intermediate consumption inputs, which may be the result of the substitution of firm-level capital investments by leasing. It is therefore possible to follow the evolution of these processes from a macroeconomic perspective, i.e. at the aggregate level and at the level of NACE 2 sectors adopted in KLEMS productivity accounting (A34 and intermediate aggregations). A potential sub-decomposition of the contribution of the intermediate consumption into sub-contributions of the three categories of intermediate consumption used in KLEMS productivity accounting, i.e. energy, materials and services, would ultimately enable even deeper analyses of these processes.

If the data on intermediate consumption and gross output are of adequate quality and the tool effects entailing the need for additional calculations are minor, the additional procedure used in the decomposition of the relative growth of gross output can bring important analytical benefits. This includes the possibility of monitoring the development of outsourcing (i.e. the main mechanism of the substitution of labour by intermediate consumption) and in the area of an unclear boundary between capital investment and intermediate consumption, i.e. frequently changing accountancy and tax regulations, and the fact that the revenue administration's activity varies to a great degree in terms of stringency, but above all, in the area of leasing (i.e. the main mechanism of the substitution of capital by intermediate consumption).

Figure 8. MFP contribution to GVA growth straightforward vs. MFP contribution to GO growth adjusted to GVA growth



Source: author's work based on data from https://stat.gov.pl/en/experimental-statistics/klems-economic-productivity-accounts/methodology-of-decomposition-in-klems-productivity-accounts-for-the-polish-economy,2,1.html.

The comparison between the two methods of calculating MFP contribution (Figure 8) shows how the substitution between production factor inputs and intermediate consumption inputs takes place over time. It can be concluded that since 2011 this substitution has discontinued, which indicates a certain stabilisation in the sphere of economic transformations related mainly to outsourcing, leasing and to some extent to the inconsistent treatment of capital investments. The reduction in the dynamics of these transformations in 2009 can be linked to the world financial crisis which occurred at that time.

The special merit of KLEMS productivity accounting is that the aforementioned analysis can be carried out by going down to the sectoral (industry) level, i.e. to lower KLEMS aggregations (A34 and intermediate aggregations). This allows for a more precise location of the transformations according to the various NACE 2 economic activities and their relevant interpretation. As it turns out (Kotlewski & Błażej, 2021), the main contributor to the difference observed in Figure 8 is section C (according to the NACE 2 classification). In section J, on the other hand, these transformations in the 2015–2016 period intensified, which is undoubtedly an interesting analytical observation, but this section is not massive enough to change the results at the aggregate level presented in Figure 8. Unusual behaviour is also observed for those sections of NACE 2 not belonging to the market economy (according to the definition of this entity adopted in KLEMS productivity accounting), i.e. L, O, P and Q.

Chapter 3
Regional productivity accounting perspective

There is no doubt that the analytical benefits gained through the performance of KLEMS productivity accounting could be additionally enhanced by carrying it out not only for the whole national economy, but also for selected regions of the country (divided into A34 and intermediate aggregations) – in Poland these would in practice mean voivodships. However, the statistical data relating to regions are insufficient in the present form, which is also a problem faced by almost all countries in the world. The basic work connected with performing productivity accounting by voivodship consists therefore in finding an appropriate method of estimating the missing data.

As the methodology for this estimation is of fundamental importance, it will be discussed as first. Subsequently, the decomposition performed in the framework of the first and second editions of the Technical Assistance Operational Programme of the EU will be presented. Although simplified in form in relation to KLEMS productivity accounting, this decomposition constitutes a kind of a bridge on the road to the implementation of KLEMS productivity accounting at the regional level, and has already provided very interesting results for economic analyses. The methodological work discussed in this chapter is therefore of basic importance to the further work on KLEMS regional productivity accounting presented in Chapter 4.

3.1. Methodology of acquiring missing data

A selection of the missing data estimation procedures have already been applied in the calculations performed for the KLEMS productivity accounting for the whole Polish economy. For example, in the previous chapter it was demonstrated how the ICT capital was estimated and how capital aggregates were decomposed into KLEMS aggregates (A34 and intermediate aggregations). The same chapter also presented the method for processing the labour factor, which allowed the assessment of the contribution of labour quality to the relative growth of GVA.

These methods are used in regional productivity accounting to the greatest extent, therefore this chapter will discuss them systematically. This approach was adopted, because, while at the aggregate level not using these methods for estimating missing data would enable implementing KLEMS productivity accounting only in a more simplified way, deviating from the standard methodology, not using them for regional productivity accounting would make its implementation impossible.

Much attention in the literature has been devoted to the division of annual aggregations into quarterly or monthly sub-aggregations. Their methodology refers primarily to the works of Chow and Lin (1971). On the other hand, the methods for disaggregating national aggregations into regional ones are extremely rarely discussed – from the point of view of the very method of making this disaggregation, not from the point of view of the collection and substantive analysis of regional data (Bordignon & Di Fonzo, 1992). Separate, sector-specific methodologies are developed, particularly often for agriculture, and also using satellite observation for data collection. These methods (except for the last one, related to specific technological progress), are often adapted by statisticians to serve a specific practical purpose. The method presented here was also developed anew for decomposition accountings carried out by Statistics Poland (Kotlewski, 2017b, 2019).

Although techniques for estimating missing data are of a strictly quantitative nature (they are specific mathematical algorithms), the decision on the choice of the estimation method, involving the selection of a statistically available proportional structure, is qualitative. An expert dealing with this issue decides whether a certain structure of statistically available data is appropriate, as it carries implicit information about the necessary, yet missing data – information that can be extracted or rather transferred from one data to another data due to their implicit proportionality.

Limitations in acquiring appropriate data very often result from the fact they are only available in inappropriate (from the point of view of KLEMS-type and other kinds of decomposition accounting) prices. There are cases where data of appropriate detail are available for other economic measures than those required by a given decomposition accounting, although being similar or substantively related. This situation can be recognised and taken advantage of by a researcher involved in estimating missing data.

Through the use of relevant methods, therefore, it is quite often possible to make recalculations or estimations that enable the preparation of data suitable for use in decomposition accounting inspired by Solow's original idea. This is necessary because, although it is quite common to refer to decomposition accounting in such a way as to recognise the mathematical elegance of the underlying model and the unquestionable objectivity of the results obtained *ex post* (based on empirical data), the latter advantage is also a weakness of these kinds of accounting, as they require access to resources of considerably detailed data, which cannot always be initially provided (Domański, 2006, 2012).

3.1.1. Historical and current prices

The system of Polish official statistics is formed by regional statistical offices with Statistics Poland at the top. They collect data on the basis of the survey program of official statistics (Program badań statystycznych statystyki publicznej – PBSSP), established annually by the Council of Ministers in a regulation which also indicates the scope of data to be collected and the entities obliged to provide them. The PBSSP is a list of all statistical surveys of official statistics conducted in a given year, including those aimed at obtaining economic data relevant to the subject of the present work.

The specificity of these economic data is that they are very often expressed in monetary units. Exceptions include data collected through some labour market surveys on employment expressed in other units, such as the number of employed persons, the number of working persons, the number of full-time equivalent jobs or the number of working hours (in the form of hours paid or hours worked). But these units too, in many applications, when multiplied by such quantities as remuneration rates, ultimately take the form of values expressed in monetary units. This becomes even more obvious in the case of surveys on goods – in many applications the sheer number of goods is multiplied by their prices, as it is only through this conversion that comparable monetary values can be obtained. In this case the price is at the same time the unit weight of the commodity in the economy.

The economic entities indicated in the PBSSP regulation, e.g. enterprises, are obliged to fill in specially prepared statistical forms (often very detailed). These reports are then submitted to statistical offices and Statistics Poland in the manner specified in the PBSSP, and compiled there, as are data from representative surveys conducted on a purposefully selected sample of business entities.[87]

If the compiled data refer to values expressed in monetary units, then they are expressed at historical prices, irrespective of the technique used to compile them. The data are collected directly from the economic environment by means available to official statistics, and are assumed not to be subject to any manipulation. From the point of view of the law (adopted by the above-mentioned public body) this is a great advantage, but on the other hand, very often such data do not fulfil the requirements of economic analyses, let alone the methodological requirements, and they usually need further processing, e.g. into data expressed in current prices or different kinds of constant prices. It should be noted that in many cases, the historical prices tend to be equal to the current prices, e.g. when the data relate to one period only. However, the current price values may in this case include revisions and adjustments, i.e. they may go beyond the scope of the data expressed in historical prices.

[87] This is a small sample of all entities, but selected to reflect structurally the whole collective as closely as possible.

A more important difference between data expressed in historical prices and data expressed in current prices can be observed in the case of data concerning accumulated values over many periods (usually yearly periods). This particularly refers to the determination of the value of productive capital, essentially represented by the stock of fixed capital in the economy[88] at the aggregate level and at various lower levels of aggregation (e.g. A34 aggregation in KLEMS productivity accounting). The data on the stocks of fixed capital expressed in empirically objective historical prices have the character of data expressed in mixed prices, i.e. coming from different years. Since it is not possible to determine the amount of capital in the economy at any given moment with sufficient precision by means of ongoing market observation, this is done by means of the perpetual inventory method. This method consists in assuming (usually in a quite arbitrary way) some initial values for the base year, adding further investments from each subsequent year, and depreciating the value of fixed assets accumulated up to that point. When the volume of capital is measured in the prices of a given year, the prices of successive years are used, e.g. to the initial value of the capital stock of the year 2000 expressed in the 2000 prices, one adds investments of the year 2001 expressed in the 2001 prices, the investments of the year 2002 expressed in the 2002 prices, and so on, up to the most recent year for which data are available. In this way, one obtains a sum expressed in mixed prices from different years. The advantage of data thus obtained is their empirical objectivity, since only the initial value of the capital is artificially generated by making certain assumptions. However, the share of this initial capital in the total capital usually is minor.[89]

Still, the values of capital expressed in 'register' prices, i.e. historical prices, differ significantly from the values of capital provided in current prices. If there is a need for the value of the capital in the economy in, e.g., the year 2015, appropriate conversions of the data from registered (historical) prices to current prices have to be made. Considering further the above example, the capital stock registered in the year 2000, expressed in the 2000 prices, and the values of subsequent investments registered in the 2001 prices, then in the 2002 prices, and so on, have to be individually converted into values expressed in the 2015 prices and only then added together to obtain the value of fixed assets, i.e. the capital in the 2015 prices (in addition, these successive

[88] The measurement of fixed capital is usually the primary means of determining the amount of productive capital in different types of economic growth accounting, including KLEMS productivity accounting.

[89] There are three ways of measuring the amount of capital in the economy: 1. the observation of market transactions (including stock market transactions) concerning capital entities; 2. the observation of insurance contracts related to capital entities; 3. the continuous inventory method (see Berlemann & Wesselhöft, 2014). The first method is subject to uncertainty resulting from high fluctuations of prices on the capital goods market. The disadvantage of the second is that insurance holders often fail to declare the true value of the insured goods. This leaves the perpetual inventory method as the most reliable, although each of the three methods of obtaining the relevant data should in ideal case produce the same results.

tranches of investments must be depreciated annually, which also requires making certain assumptions, as discussed further).

In the light of the argument above, this means that data on the volume of capital in current prices do not have the value of direct empirical objectivity of directly compiled data, but have been generated by certain accounting operations that are inevitably subject to approximation errors connected with methods of accounting for price inflation, which are to some extent controversial. Capital goods inflation is different from the Consumer Price Index (CPI) inflation, including the Harmonized Index of Consumer Prices (HICP) inflation. Inflation differentiation occurs also in the case of different varieties of capital and according to different economic activities, e.g. according to NACE 2. This problem is to a large extent overcome by disaggregating the economy into sections and divisions (and possibly further into smaller aggregations), carried out in Leontief's IOTs and SUTs as well as by dividing the capital at different levels of aggregation into its different types.

One more issue is important here. In order for inflation accountings to be absolutely accurate and representative, the baskets of goods that enter inflation accountings must always be identical in relative terms, that is, the shares (weights) of the representative goods in the entire basket of goods should be identical. For example, the comparison of inflation in Germany with inflation in Poland would be accurate only if an identical representative basket of goods were used in both countries. However, German statisticians use a different basket of goods to calculate inflation in Germany than Polish statisticians for inflation in Poland – this also applies to inflation related to capital goods, whose prices are subject to larger fluctuations than prices of consumer goods. The use of a common basket of goods does not provide a conclusive solution either, because when comparing inflation in two countries of different sizes, the common weighted basket of goods reflects in fact the basket of goods of the larger country, and the inflation calculated this way may be very different from the actual inflation in the smaller country.

By analogy, the inflation rate in a given country differs from inflation rates in its individual provinces or cities, e.g. the inflation rate in Warsaw is certainly different than the inflation rate in the whole Poland. In other words, the inflation calculated for a given country is an abstract which does not strictly apply to any specific location. Fortunately, national markets are integrated enough to make these differences not too large to preclude any analyses (especially in relation to consumer goods, whose prices show less spatial variability than prices of capital goods, particularly when real estate is involved). However, when carrying out such analyses, these inaccuracies should be taken into account so as not to draw overly far-reaching analytical conclusions.

The issue of the incomplete comparability of baskets of goods arises not only in the spatial dimension (i.e. between different countries or even regions of a given country), but also in the time dimension (i.e. between time periods in a given country or even a given location). The baskets of goods used to calculate inflation (and especially the weights of the representative goods in the baskets) change each period and may become difficult to compare over time.[90] For this reason, data on the size of capital in current prices are somewhat immanently deprived of the value of absolute empirical objectivity, as they are the result of approximate estimation methods adopted by practitioners, which may additionally vary across countries (the volatility of prices of capital goods is greater than the volatility of the prices of consumer goods both in time and space). To remedy this problem to at least some extent, the calculation methods used by Eurostat, the OECD and other international statistical organisations are harmonised – after some compromise solutions having been adopted.

3.1.2. Current and constant prices

A similar problem concerns the conversion of current prices into constant prices. The data collected from the last year in current prices are converted into data expressed in the prices of the previous year. For cumulative values from a number of years – as in the case of capital, which is considered cumulative investment – they are added to the total present value calculated for the previous year. For non-cumulative values, such as GDP, the total present value is converted into the value expressed in the prices of the previous year.

When comparing the value shares of certain products in the economy or the value of particular kinds of capital in the value of the entire capital, it is sufficient to use current prices. Conversion into constant prices is not necessary, and may even distort the results – if the prices of certain goods rose relatively faster than others, conversion into constant prices would produce false (i.e. not reflecting the actual condition) shares in current economic aggregates. This is particularly important in the case of capital goods, which are prone to high price volatility (impacted by the business cycle) and volatility of the demand for different kinds of capital connected with the evolution of the needs of the economy (as already mentioned, this volatility is greater than that relating to consumer goods), which also results in price volatility. Significant price volatility also applies to investment goods, i.e. new capital goods.

[90] Probably the best solution to this problem is to use common baskets of goods for adjacent periods, e.g. one-year periods. For multi-year periods, a compound percentage method can be used for consecutive pairs of periods. This chaining procedure gives slightly different results than the simpler procedure of directly comparing the prices of two distant annual periods (which requires the adoption of a common basket of goods for these distant periods which differ significantly). If the multi-year periods are short, common baskets can be used for following consecutive years. This issue is very complex and will not be developed here in accounting terms (see Diewert, 1976, 1978, 2004; Fisher, 1922; IMF, 2004; Milana, 2009; Schreyer, 2004).

Using current prices only, however, does not allow a distinction between real annual increases in the quantity of goods produced and the price inflation of these goods. This issue is important for such flows as GDP, GVA or gross output, and is even more significant to stocks, including the stock of fixed capital (i.e. the stock of physical capital). For example, the annual inflation of all goods in the entire economy may stand at 2%, and for the distinguished capital and investment goods – 8%. In this situation, a 6% increase in the investment in these goods at the current prices could in fact mean a real decline in the investment. This situation may occur when investment goods become more expensive due to the lower availability of their components. More often, however, these circumstances take place because of the fluctuating market demand for these goods. In business cycle upturns, the demand for capital and investment goods can increase so much that, in addition to their increase in the stock of new real investment, their prices increase, usually at a much faster pace than the average cyclical price increase in the economy.

In the light of the commonly accepted theory – in line with the concept of the net present value of capital as a flow of future incomes discounted by an interest rate[91] – it is assumed that the real value of capital and investment goods should reflect their productive capacity now and in the future. Considering the above argument, however, it becomes clear that the current prices of these goods will not reflect the increase in their real value correctly, since they also include inflationary price increases that combine monetary (relating to the entire economy), cyclical (resulting from the increasing demand for a given capital or investment good) and cost (specific to a given investment good, connected with its production process) effects.

Thus, when calculating growths and dynamics, it becomes necessary to convert the already accounting-generated current prices into constant prices (which are therefore also accounting-generated abstracts). This is an indispensable procedure both for the flows (e.g. GDP, GVA, gross output) and for the resources (e.g. the stock of fixed assets, i.e. accumulated capital goods). In the latter case it is even more *de rigueur*.

Thus far in the discussion, constant prices have been understood as values expressed in the prices of the previous year. However, when comparing data from multiannual periods, the use of constant prices from the previous year in some situations becomes insufficient. For example, if instead of the values of the successive years of the period 2010–2015 initially expressed in current prices, we use the constant prices from the successive years of the period 2009–2014, the problem of the incomparability of data from many years remains unresolved. Therefore, it is sometimes also necessary to convert the data into common constant prices of a chosen base year, e.g. 2005 or 2010. Usually the base year is periodically moved forward in time.

[91] According to the NPV methodology.

According to the currently preferred methodology, initially suggested by Diewert (1978), when converting data into figures expressed in constant base year prices – in order to minimise errors – chain conversions are used, involving the accumulation of changes from adjacent annual periods rather than direct comparisons between more or less distant annual periods and the base year period. As a result, the data obtained for lower aggregations do not exactly add up to the values obtained for higher aggregations. In practice, after the recalculation of data for individual economic domains and for the entire economy, the new data values for economic domains do not add up exactly to the new value calculated the same way for the entire economy (although they did before the recalculation). No mathematical method has been developed so far to overcome this problem conclusively, so the data recalculated this way are subject to some tool deviations from the unknown true values – the longer the chain of recalculations, the greater the tool deviation. For this reason, it is necessary to periodically shift the base year, e.g. from 2005 to 2010 and then to 2015, etc., unless the results of analytical inference are required only as qualitative approximations.

3.1.3. Capital assets before and after depreciation

With regard to the values accumulated over many years, other processes may take place leading to even deeper data differentiation. In the case of capital goods, the phenomenon of capital depreciation exists, in the framework of which the gross values for capital must be depreciated by the size of this depreciation in order to obtain the corresponding net values (customarily, for the capital, the term 'gross' was adopted to describe data before undergoing this procedure and the 'net' term – after its completion). Using the perpetual inventory method, the calculated values at current prices and constant prices must be converted into depreciated values, which means that they have to be reduced as the ageing capital loses its value over time. It may be assumed here that the reason behind this situation is that the accumulated value of the flow of future income resulting from this capital, discounted at a given interest rate, decreases over time. In effect, individual capital goods become unproductive or even loss-generating, which necessitates their liquidation.

Capital depreciation is a complex process involving numerous sub-processes, such as the gradual decline in the efficiency of capital equipment, its increasing failure rate (with the possibility of premature removal due to a major breakdown or accident), its ultimate liquidation at the end of its useful service life, or liquidation owing to the unexpected arrival of novel equipment. The complexity of this phenomenon results in the adoption of a number of simplifying assumptions. It is usually done by subtracting each year a certain portion of the value of capital from its total value expressed in current or constant prices. For the purposes of productivity accounting of the KLEMS and other types, this operation must definitely be carried out regardless of the above-mentioned conversion of inventory (historical) prices into current prices and then into constant prices.

The easiest way to carry out this operation is by means of linear accounting depreciation. It involves, for example, deducting each year from the initial value of 100 in certain units for a certain hypothetical (here assumed) capital good, say, 10 of these units of value, so that in the subsequent years 90, 80, etc. are left. However, the economic theory (the perpetual inventory method[92]) in principle requires that capital depreciation be modelled geometrically rather than linearly, as conceptually discussed in more detail in Chapter 1. The depreciation write-offs should be a fixed proportion of the present value of that capital, not a fixed proportion of its initial value. All generations of capital (vintages of capital) should be therefore depreciated successively each year based on their actual value. A certain phenomenon, modelled in Chapter 1, proves helpful here. It turns out that linear depreciation performed at the level disaggregated to different kinds of capital for sections and divisions of the economy and other classification groupings, as well as for specific generations of capital, produces results consistent with the geometric depreciation for relevant higher aggregations, if the appropriate depreciation rate is adopted for the latter. However, one has to bear in mind the occurrence of tool deviations relating to the cumulative (compound) chain recalculation of data.

3.1.4. Stocks and flows

The determination of the initial total economic value, e.g. the initial stock of fixed assets, prior to the launch of the perpetual inventory procedure for newly brought to life tranches of capital resulting from an investment, may be subject to a significant deviation from the real but unknown value. This is often the case when dealing with a resource such as a stock of fixed assets. These deviations can sometimes be significant also for the 'flows'.

For example, the register value of GDP for the entire economy does not include the shadow economy (nor, obviously, the black market). Of course, there are also other reasons for which the GDP reference value may include some deviation from the real value. This deviation is compensated for by appropriate re-estimation, which is also not perfect, as it is often based on the subjective opinion of experts. Therefore, the 'levels', i.e. the calculated absolute quantities for resources and certain flows, may sometimes include quite substantial deviations from the unknown real values. Statistical practice shows that they are usually relatively larger when the levels concern resources (e.g. capital stock) and smaller in relation to flows (e.g. GDP).

[92] For current principles and a systematic analysis of this method, see Berlemann and Wesselhöft (2014).

However, statisticians are often interested in examining the increments of specific values only, and this can apply to both stocks (resources) and flows. Observations show that for this type of data deviations from the real values are very often much smaller (also in relative terms). To illustrate this, let us use a hypothetical example: if for a certain economic quantity a level of 60 is obtained and its real unknown value is 80, then, at the same time, it is very often observed to have increased to the value of e.g. 63.9, while the real unknown increase has reached the value of 84. It therefore becomes clear that in this case the increment has a smaller relative deviation from the unknown real value than the level (also when an increase is expressed in percentage or percentage points).

The above is particularly often and clearly observed when, for a given quantity, the level is a stock and the increment is a flow (e.g. when the level is the present value of capital and the flow is the current investment). This is also the case for an increment in the capital stock equalling the difference between the investment and the capital depreciation. For in this case, the present value of capital (i.e. in principle, the value of the stock of fixed assets) must be estimated by means of complicated accountings, while current investments require observation only.

This also frequently applies to situations, where both the level and the increment are flows. The volume of investment, for example, is usually measured less accurately than its increment in relative terms. The systematic error in the measurement of investments is usually reproduced and therefore almost disappears when measuring their increment. Caution is needed, however, because a random error (outlier) may, in relative terms, affect the level only slightly, whereas the increment – very significantly. An outlier may in turn be a real fact at the level of lower aggregations (e.g. some investment may be the first capital brought to life at the level of this aggregation). Therefore, the empirical observation of individual economic events also proves useful.

Experience shows that deviations from the unknown true values in the relative shares of components in the case of 'structures' also tend to be significantly smaller than in the case of absolute levels. Let us use a hypothetical example to illustrate this issue: suppose that the value of a certain aggregate of 60 units is divided into components A, B, C, D and E. The value of component A is reported to be six units, or relatively 10%, the value of component B is reported to be 12 units, or relatively 20%, the value of component C is reported to be 15 units, or relatively 25%, etc. In this situation, assuming the same percentages for the real but unknown value of 80 will usually result in very small deviations from the real values, since the structures are generally more stable and reliable than the levels.

Suppose, then, that the true value of 80 is known to us, but only in aggregate form (since the unknown aggregate values are often easier to estimate), and that the value amounting to 60 observed by imperfect data collection techniques is also available broken down into shares of its components A, B, C, D and E (since a particular limited observation can be deepened). It is then possible – with a fairly high probability of not making a significant error – to transfer the known structure for the value of 60 proportionally to the value of 80, i.e. to re-estimate component A from 6 to 8 units, component B from 12 to 16 units, component C from 15 to 20 units, etc. This transfer of structure is often subject to lower deviation from the real values than the results of an additional direct empirical examination of individual levels (and also of some flows) to determine the whole real structure. Because the data on aggregate levels (and some aggregate flows) are often of better quality (thus subject to lower deviations from true values) than the data on the levels (and some flows) at lower aggregations, the methods of disaggregating the aggregate data to lower aggregations (involving structure transfer) can often deliver better results (closer to the unknown true values) than some direct surveys and other additional observations.

The operations described above can also be used to transfer structures available for data in one kind of prices to data in other kinds of prices. For example, data on the value of capital by NACE section and by voivodship are available for the Polish economy only at register (historical) prices, and the data for these values of capital in other prices are available only for the whole country. It is therefore possible to use the known structure at the aggregate level and at the level of lower aggregations in register (historical) prices, to distribute the known aggregates in other prices (e.g. current or constant prices) into lower aggregations such as individual voivodships or sections of NACE. If the data which is to be re-estimated concern the increments rather than the levels, the relative deviations from the real values are usually even smaller.

However, the use of appropriate structures requires certain expertise, either acquired through experience in working with data or from knowing the theory – the decision to use the appropriate structure for disaggregation is a qualitative activity. The methods of disaggregation can therefore be of an innovative character in relation to a particular study, which is also the case with the algorithms shown below.

3.1.5. Algorithm apparatus used to assess missing data

The fact that some data may be available only in register (historical) prices concerns first of all fixed assets in the database of the Local Data Bank (Polish: Bank Danych Lokalnych – BDL) of Statistics Poland. These are gross fixed assets, i.e. they do not include capital depreciation. Data on net fixed assets (after depreciation) in current prices by NACE section are available in Statistics Poland's data repositories, but without a subdivision into regional aggregations, i.e. voivodships. It is these data that are appropriate for further use, but they lack this regional breakdown. However, by means of a certain mathematical procedure, it is possible to transfer the structure concerning the spatial distribution of fixed assets by voivodship included in the data from BDL to the above-mentioned data in current prices. For this purpose, the following equation can be used:[93]

$$KN_{BSW} = \frac{KB_{ESW}}{KB_{ES}} KN_{BS}, \tag{56}$$

where KN_{BSW} denotes the calculated net fixed assets (or net capital KN, i.e. after depreciation), in current prices B, according to NACE sections S and voivodships W (so it is a table, or data matrix of $S \times W$ dimensions), KB_{ESW} – gross fixed assets (or gross capital KB, i.e. before depreciation), in register (historical) prices E, according to NACE sections S and voivodships W (these data come from the BDL repository; it is also a table, i.e. a data matrix of $S \times W$ dimensions), KB_{ES} – gross fixed assets (or gross capital KB), in register prices E, according to NACE sections S, for the entire Polish economy (this is a vector of data taken from the BDL of S dimension), and KN_{BS} – net fixed assets (or net capital KN), in current prices B, by NACE section S, for the entire Polish economy (this is a vector of data of S dimension from tables compiled for the purpose of data transmission to Eurostat – these data are also available at the level of NACE divisions and groups of divisions, i.e. at the level of A64 aggregations required by Eurostat in the TTs.

If the time dimension is taken into account, i.e. subscript t is introduced into equation (56) (as in many previous equations), then all data vectors become data tables (data matrices) and data tables become spatial data 'cubes'. The operation, whose algorithm is expressed in equation (56), must be performed at the level of each cell of the aforementioned tables and 'cubes'.

At this point it seems necessary to draw attention to a specific circumstance. At the moment of performing the factor decomposition accounting described in this chapter, the sectoral division according to the NACE 1 and NACE 2 classification systems applied on the BDL platform was the division into A12 aggregations, i.e. into 12 groups of sections and sections common to both systems, as in Table 4.[94]

[93] As far as possible, throughout this work we have kept the symbols from the original sources (here: Kotlewski, 2019).

[94] At present, in the BDL data repository, other than A12 aggregations are sometimes used (not necessarily more subtle). However, these evolutionary changes are not essential for the present discussion (from a conceptual point of view).

Table 4. Correspondence between common A12 aggregations from the NACE 1 and NACE 2 classification systems in the BDL data repository

NACE	Aggregations										
1	A, B	C, D, E	F	G	H	I	J	K	L	M	N	O
2	A	B, C, D, E	F	G	I	H, J	K	L, M, N	O	P	Q	R, S

Source: author's work based on BDL.

This was therefore a less subtle subdivision than the simplified correspondence between the NACE 1 and NACE 2 systems used in KLEMS productivity accounting, where a subdivision into A14 aggregations rather than A12 aggregations is used for the same purpose. In KLEMS productivity accounting, aggregation C, D, E in the NACE 1 system is split into separate aggregations C, D and E, and aggregation B, C, D, E in the NACE 2 system is split into separate aggregations B, C and D, E. This is quite a significant difference due to the fact that section C (according to the NACE 2 classification) plays a very important role in economic analyses. When circumstances allowing the verification and update of the accounting of this factor decomposition occur, its revision, consisting also in replacing A12 aggregations with A14 aggregations, will enhance the value of these accountings for further analyses. It also follows that all the formulae proposed in this subsection should be subscribed with the symbol $A12 \in BDL$ (data for A12 aggregations belonging to the BDL data repository) by analogy with formulae (32)–(36). However, these formulae use different subdivisions by aggregations A64, A34 and A14, and the formulae in this subsection use only one subdivision by aggregation A12, so instead of the composite symbol, only subscript S is used (to denote sections and groups of sections).

In formula (56), the basic premise adopted to justify this operation is the very probable occurrence of the fact that, in general, the structures related to the spatial distribution of the aggregate and cumulative investments between voivodships did not change significantly over time (because generally the structures change slowly, which has already been explained), or at least in the period under study, i.e. after 2001, when high inflation in the economy subsided (this fact occurred in the Polish economy at the beginning of the 21st century). It is, after all, a feature of register (historical) prices that they overestimate the importance of later tranches of capital in comparison with the older ones, as they contribute to giving the former a greater weight depending on the rate of price change.

The structure transferred from data in register (historical) prices from the BDL data repository onto data in current prices are therefore also characterised by this feature, but this situation is better (for economic analyses in which references to the present are more important) than if it were the other way round (i.e. if the weights of older capital tranches were relatively increased in relation to the later ones, in comparison with the unknown real situation). In situations where adequate data on the stocks of fixed capital in current prices by voivodship are not available, carrying out this operation seems justified, and its effects in the form of deviation from the unknown true values will be limited, especially when the increments and not levels (as explained above) are the subjects of the study.

Data on net fixed assets (net capital, i.e. after depreciation) in current prices may not be sufficient in all situations involving relevant calculations within the productivity accounting framework (as mentioned earlier). In addition, the values for net fixed assets have to be obtained also in constant prices. For this purpose, the following equation has been used:

$$KN_{SSW} = \frac{KN_{SS}}{KN_{BS}} KN_{BSW}. \tag{57}$$

In this equation, the calculated value of net fixed assets (net capital KN, i.e. after depreciation), in current prices KN_{BSW}, from equation (56) was multiplied by ratio KN_{SS}/KN_{BS}, where KN_{SS} stands for net fixed assets in constant prices (first subscript S) by NACE section (second subscript S) for the entire Polish economy (data taken from TTs – also available at the level of the A64 aggregation, that is, at the level of NACE divisions), and KN_{BS} – net fixed assets in current prices for the Polish economy (the value used earlier in equation (56) – this is also the data from TTs available at the level of the A64 aggregation). This operation allows obtaining KN_{SSW}, i.e. the calculated net fixed assets in constant prices (the first S subscript), by NACE section and voivodship (denoted by appropriate subscripts used earlier). In both equation (56) and (57), all values with two subscripts are data vectors, and values with three subscripts are data tables (matrices). Here too, data vectors and data tables turn into data tables and data 'cubes' if one introduces the time dimension into equation (57) in the form of additional subscript t; here omitted for simplicity.

In this process, a seemingly strong assumption is made that the price inflation in relation to capital and investment goods follows a similar pattern both at the level of each voivodship and at the level of the national economy as a whole. There are several reasons for this assumption. On the basis of the experience gained when working with data, it can be presumed that the differences between voivodships in terms of the inflation of capital and investment goods result, to a large extent, from a different sectoral (industry) composition of these goods, i.e. a different structure of these goods by voivodship. This differentiation was taken into account in equation (57), because the estimation operation according to this equation is performed at the level of individual sectors (industries) of the economy (i.e. sections of NACE). The deviations from unknown true values were thus significantly reduced.

The second premise results from the fact that inflation is an increase in prices, it is not the price level. Although the relative price levels of capital and investment goods (established for a rather longer period of time) differ to some extent between voivodships, their relative change is usually similar, because it results from other economic reasons than from a relatively permanent differentiation of price levels of capital and investment goods between voivodships, resulting, for example, from a different level of their economic development (prices of capital and investment goods, as well as other prices are usually higher in more economically developed regions).

Another premise is that the regional variation in prices of investment goods, i.e. new capital goods, is smaller than that of old capital goods, whose prices on the other hand are held at certain levels by the prices of initial investment outlays. Because of the law of one price and because of similar production costs, many new capital goods, i.e. investment goods, must have a similar price. This counteracts large price divergences, especially for mobile capital goods (such as machinery and equipment) or means of transport (e.g. cars). It is therefore beyond doubt that the relative increase or decrease in prices of mobile capital goods, especially new investment goods, follow a similar pattern in all voivodships (otherwise temporary streams of trade in these goods would occur, undeveloped by large economic agents[95]). The only problem is the delayed spatial propagation of the cyclical fluctuations in the prices of capital goods and the fact that these fluctuations tend to be larger in regions that are growing faster.

In productivity accounting, in addition to the data for the stocks of capital, certain variables concerning flows at constant prices are also necessary (in order to free the changes in their values from inflationary effects and obtain real increments). These flows are the GVA, labour remuneration (compensation) and capital remuneration. In TTs, the data for these values are given by NACE section of PKD for the entire Polish economy, in current prices and constant prices. However, these data are not broken down by voivodship and therefore they served only as a structure for estimating the GVA in constant prices by voivodship. For this purpose, the following equation was used:

$$WDB_{SSW} = \frac{WDB_{SS}}{WDB_{BS}} WDB_{BSW}, \tag{58}$$

[95] Such temporary flows of trade may sometimes arise for certain specific goods, but this is linked to circumstances that can be regarded as pathological in the functioning of the economy and concerns a small number of goods, usually consumer goods.

where WDB_{SSW} denotes the calculated GVA at constant prices (subscripts have the same meaning as in the previous formulae), according to NACE sections and by voivodship (this is a table, i.e. a data matrix), WDB_{SS} – GVA at constant prices according to NACE sections for the entire Polish economy (this is a vector of data taken from the TTs[96] – data also available at the level of A64 aggregations required by Eurostat), WDB_{BS} – GVA at current prices according to NACE sections for the entire Polish economy (this is also a vector of data from the same source;[97] data available also at the level of A64 aggregations), WDB_{BSW} – GVA at current prices by NACE section and by voivodship (this is a data matrix taken from the BDL data repository). Taking into account the time dimension, i.e. also the omitted for simplicity t-subscription, would make all vectors become tables (matrices), and all tables (matrices) – 'cubes' of data, which is accompanied by the application of equation (58) at the level of each cell.

The applicability of equation (58) is based on the assumption that individual voivodships' inflation is similar to the inflation for the entire Polish economy at the level of individual sectors (industries), namely, e.g. sections of NACE. This assumption has, as it seems, sound foundations. A part of the goods of which GVA is a component in the total GVA for the country's entire economy are investment goods. As already mentioned, the variation of their prices in the geographical economic space, especially at the level of one country, is smaller than the variation of prices of old capital goods. In turn, the value of the latter is not a component of the GVA (unlike the former), so the more price differentiated capital goods are not included in equation (58).

For investment goods, and especially for the remaining consumer goods and services, and possibly others (e.g. military or medical), the variation in inflation between voivodships is largely the result of a different structure of their production by region, which was largely taken into account in equation (58). In terms of individual goods, this price variation is much smaller – goods of a specific kind are produced at similar prices or at least the price relations between similar goods remain relatively rigid for the whole country. To some extent this is also the case for selected aggregations according to the NACE division lines. Thus, since equation (58) is calculated independently for individual NACE sections, the deviations in the values calculated in the equation from the unknown true values should be considered as much reduced.

The circumstance strongly supporting the assumption adopted in equation (58) is also the fact that price inflation of goods concerns price increments, and not price

[96] The BDL data repository can also be used for this purpose, as these data can be found in both sources. However, data from TTs are made using a methodology closer to the SNA or ESA rules.
[97] As above.

levels. Voivodships may differ in price levels of various goods that make up the GVA, which is, however, related to other economic reasons than the inflationary increase in prices of these goods. While the price levels of these goods vary to some extent across voivodships (although less than the price levels of capital goods), the relative change in these prices, i.e. inflation, is comparatively more uniform in terms of its spatial distribution, i.e. also between voivodships. The law of one price (which takes into account the persistent price differentials connected with discrepancies in the level of economic development of different regions, and thus voivodships) makes the spatial propagation of new prices even faster than if it were solely due to the rate of the distribution of new tranches of goods.

TTs provide constant and current prices for GVA. However, in the case of labour remuneration (compensation), data are only available in current prices in these tables. Inflation for GVA, varies to some extent across sectors of the economy, i.e. also across NACE sections (as prices of different products and services may change at various rates). In contrast, it is assumed that the inflation of the prices of products and services in relation to the labour market should be treated as a weighted average of different inflations from a number of sectors, as workers from particular sectors are consumers of a broad basket of goods and services from many sections. In order to estimate the missing values for labour remuneration (compensation) in constant prices, an equation similar to equation (58) was used. In the equation, the structure of the differentiation by sections of GVA in constant prices relative to GVA in current prices was replaced by the ratio of this GVA in both mentioned prices for the Polish economy as a whole:

$$WP_{SSW} = \frac{WDB_S}{WDB_B} WP_{BSW}. \tag{59}$$

In this equation WP_{SSW} denotes the calculated labour remuneration (compensation) in constant prices by NACE section and by voivodship (it is a data matrix), WDB_S – GVA in constant prices for the entire Polish economy (a value taken from the tables compiled for the purpose of data transmission to Eurostat[98]), WDB_B – GVA in current prices for the entire Polish economy (a value taken from the same source), WP_{BSW} – labour remuneration (compensation) in current prices by NACE section and by voivodship (a data matrix taken from the BDL data repository). When the time dimension is taken into account and the t subscript is introduced into equation (59), the individual data change into data vectors, and the matrices – into data 'cubes'.

The omission of the diversification by section in this case is accompanied by the omission of the diversification by voivodship. Equation (59) uses the ratio of the

[98] The BDL data repository can also be used for this purpose, as this information is present in both sources.

GVA in constant prices to this value in current prices for the entire Polish economy, not for voivodships, in the estimation process, as consumers are recipients of goods and services from the entire economy rather than from one given voivodship (which is related to the fact that regional imports and exports of goods constitute more than half of the consumption and production, respectively). A differentiation by voivodship, achieved in equation (59) by using a WDB_{SW}/WDB_{BW} ratio, i.e. a ratio of GVA in constant prices by voivodship to GVA in current prices by voivodship, instead of the WDB_S/WDB_B ratio, would probably cause the results to show much greater deviations from the unknown true values.[99]

However, a certain issue arises in the case of accounts based on equation (59). The inflation of goods in relation to the labour factor should be calculated using an appropriate breakdown on the demand side, i.e. independently for the different groups of the recipients of goods and services, separately for the employed persons and the working persons, according to their profession, education, age, sex, wealth and other divisions of the labour market and consumer groups (with the level of wealth being the most important for the price level among the above-mentioned groups of the recipients of goods and services). Thus, it would be necessary to create different baskets of goods for different groups of consumers and calculate the inflation for these separate baskets of goods. However, this is currently not feasible and, to the author's best knowledge, is not done in any country (it would be an extremely innovative measure and perhaps useful in productivity accounting).

For productivity accounting, the division into NACE sections (and especially into divisions or groups of divisions, as in KLEMS productivity accounting) applied to calculate GVA, as in equation (58), to a large extent mitigates this problem (as far as its effects on the overall decomposition results are concerned), as differences in the inflation for the various groups of the recipients of goods and services result to a large extent from a different structure of their consumption. This variation, in turn, was taken into account from the supply side in the types of productivity accounting discussed in this work.

Labour remuneration (compensation) accounts are related to capital remuneration accounts in the sense that the latter is calculated as the difference between GVA and labour remuneration (compensation) at each level of aggregation. Indeed, in the SNA and ESA systems, it is assumed that, as for equation (5), the following equation applies:

$$WDB = WP + WK, \tag{60}$$

[99] The WDB_{SW} variable, i.e. GVA at constant prices by voivodship, can be calculated as the sum of the values from the left-hand side of equation (58) and the WDB_{BW} variable is the value that can be retrieved from the BDL data repository.

where WDB denotes GVA at each level of aggregation,[100] i.e. also by section, by voivodship and by section and voivodship at the same time. Similarly, WP and WK mean labour remuneration and capital remuneration at each of the above-mentioned levels of aggregation. This also applies to real values, i.e. in the case of the real remuneration of capital, understood as its value in constant prices, equation (60) transformed to the following form can be used to estimate the respective values at the level of the lowest aggregations used in the accountings:

$$WK_{SSW} = WDB_{SSW} - WP_{SSW}. \tag{61}$$

In equation (61) the values from equation (60) are subscripted according to the convention adopted here, so that they refer to values in constant prices, by NACE section and by voivodship. Despite their simplicity, equations (60) and (61) are not trivial, as they are strictly valid only under the assumption of perfect competition operating in the economy in the light of the economic growth theory. Moreover, their further application in economic productivity accounting connected with Solow's decomposition or the decompositions used within the KLEMS productivity accounting framework requires the theoretical assumption that there are constant returns to scale in the economy, which was discussed in Chapter 1.

The issue that must be addressed in the process of implementing productivity accounting at the regional level involves finding a way to estimate certain values for the working persons in the economy, while the available statistical data usually relate to employed persons only. This problem is crucial to the introduction of KLEMS productivity accounting at the regional level, i.e. in the case of the Polish economy – at the voivodship level.

This problem does not exist either at the aggregate level for the entire Polish economy or for the NACE sections and divisions, or, consequently, for the aggregations used in KLEMS productivity accounting performed without the regional perspective (i.e. without the subdivision into voivodships), because TTs contain data on both the employed and the working persons. The difference between these categories has been unambiguously defined – the number of working persons is the total of the number of the employed and the number of self-employed persons. Similar definitions are adopted for the other values related to the 'labour' production factor which are important in productivity accounting: the number of hours worked by working persons is the total of the number of hours worked by employed persons and the number of hours worked by the self-employed; the remuneration (compensation) of working persons is the total of the remuneration (compensation) of employed

[100] As already mentioned, we have kept the symbols from the original works in order to facilitate the linkage with them.

persons and the remuneration of the self-employed. However, it is not entirely obvious that the proportions between the values for the employed and the values for working persons are identical for all the above-mentioned categories. In this situation, when some data need to be estimated, a structure based on the number of hours worked (in the light of KLEMS productivity accounting and other factor decomposition accounting theories of this kind) is preferred.

From this point of view, the data offered by the BDL repository fit for productivity accounting relate to employed persons only. The data are available by section and voivodship, also for time series adopted in the productivity accountings already performed for the Polish economy. However, data concerning working persons have very short time series and are available in the BDL data repository for groups of sections (not individual sections) too aggregated to be useful in productivity accounting. Therefore, it was necessary to convert the data on employed persons into corresponding data on working persons. This operation was carried out according to the equation:

$$P_{SW} = \frac{Z_{SW}}{Z_S} P_S, \tag{62}$$

where P_{SW} denotes the calculated number of working persons by NACE section and by voivodship, Z_{SW} – the number of employed persons by NACE section and by voivodship (data taken from BDL), Z_S – the number of employed persons by NACE section (data taken from BDL), and P_S – the number of working persons by NACE section (data taken from TTs available also according to A64 aggregations required by Eurostat, i.e. at the level of divisions or selected groups of NACE divisions). Data with one subscript are vectors and those with two subscripts refer to tables (matrices). The inclusion of the time dimension – as for several of the previous equations – turns vectors into matrices and matrices into data 'cubes'. By this means, data on the number of working persons became as available for productivity accounting as data on employed persons. The operation performed according to equation (62) allowed the determination of the labour factor stock for all the lowest aggregations for the productivity accountings available in BDL.

Applying equation (62) makes it possible to use the relative structure of the number of employed persons by voivodship (available in BDL) in the process of distributing the number of working persons available by the NACE section only for the entire Polish economy, by voivodship. The shares of the separated NACE sections in particular voivodships are different, therefore this operation differentiates voivodships in terms of the proportion between the working and employed persons.[101] For this reason, a very large part of this diversity between voivodships has been reflected in the estimated data. One can safely assume that regions, and therefore also voivodships, differ in this respect mainly for structural reasons, i.e. (according to the understanding of this notion known in economic sciences) due to a different structure in economic activities, also (and especially) according to NACE sections.

Moreover, due to other data needed in productivity accounting and related to the labour factor, it is necessary to assume that they are in the same proportion as the number of working persons to the number of employed persons. Most importantly, according to the theory relating to productivity accounting, the number of hours worked is the best measure of the labour factor resource, rather than the number of persons involved in the production process, for which equation (62) is adopted. Due to the lack of the availability of relevant data in BDL, it has become necessary to assume that the proportion of hours worked by the working persons to the number of hours worked by the employed persons does not differ much from the proportion of the number of working persons to the number of employed persons. Intuition and experience suggest that this is a sound assumption, often practised in productivity accounting in the absence of relevant data on the number of hours worked.

The second assumption often used in productivity accounting with respect to the labour factor does not seem as obvious. The point is that in the light of equation (62), it is also necessary to make the assumption that the ratio between the remuneration (compensation) of working persons and the remuneration (compensation) of the employed persons is the same as between the number of working persons and the number of employed persons at all aggregations adopted in a given accounting. A variation of this assumption, often used in productivity accounting, is the ratio between hours worked for working persons and hours worked for employed persons (instead of the ratio of the number of working persons to the number of employed persons); this variation improves the situation if appropriate data are available. When accountings are made for the whole national economy and not by voivodship, separate, slightly different proportions for hours worked are readily available, which in this case is beneficial to these accountings. In KLEMS productivity accounting performed for whole national economies, this assumption is widely practised and works well in developed Western economies, as the share of agriculture in these economies is very small. In order to reduce the possible over-reaching divergence from the unknown true values, a two-tier approach is needed at the provincial level.

[101] Within individual NACE sections this proportion is relatively uniform, also in spatial terms. The biggest differences between the number of working persons and the number of employed persons concern e.g. agriculture and services related to tourism, catering and other minor services, while the smallest – the state administration, heavy industry, sectors dominated by large companies and monopolies, banking, etc.

The first of the two further considered methods of adjusting the labour remuneration (compensation) of the employed persons to the labour remuneration (compensation) of the working persons is to use the commonly applied in productivity accounting proportion of hours worked for the working persons to the number of hours worked for the employed persons. This proportion, under the conditions of using equation (62), is by definition identical to the proportion of the number of working persons to the number of employed persons. This operation is necessary to determine the share of labour remuneration in GVA, which in productivity accounting is equated to elasticity α, according to the assumptions of perfect competition and constant returns to scale (mentioned in Chapter 1), and which, depending on whether it concerns the working persons or the employed persons, will be indexed in the accounts further presented as either α_Z or α_P (subscript Z indicates the parameter applicable in the accounts for the employed persons, and subscript P – for the working persons). In order to determine the labour remuneration values at the voivodship level, the following equation was used:

$$WP_{PSW} = \frac{H_{PS}}{H_{ZS}} WP_{ZSW}, \tag{63}$$

where WP_{PSW} denotes the calculated labour remuneration for employed persons P by NACE section and by voivodship, H_{PS} – the number of hours worked for working persons P by NACE section (data obtained from TTs also available at the level of A64 aggregations used by Eurostat), H_{ZS} – the number of hours worked for employed persons Z by NACE section (data from the same source, also available at the level of A64 aggregations), WP_{ZSW} – labour remuneration for employed persons Z by NACE section and by voivodship (data from the BDL data repository).

This operation required assuming that the self-employed pay themselves in the net operating surplus (including net mixed income) the same wage per hour worked as the employed persons receive under contracts of employment for such an hour (including social benefits). However, under the conditions of using equation (62) as a starting point for determining the ratio between the hours worked by the working persons and the hours worked by the employed persons, the somewhat less accurate assumption is made that the self-employed persons pay themselves the same *per capita* wage as the employed persons, i.e. regardless of the actual hours worked. The remainder of the mixed income is considered as the return on capital, which must additionally be included in the remuneration of capital.

These demanding assumptions about labour remuneration (compensation), which are necessary to apply equation (63), are largely plausible. The variability of wages per hour worked and also per person employed is to the greatest extent related to the type of activity performed, as outlined in NACE. Equation (63) is applied at the sector level – it can also be applied at the level of not only sections of NACE, but also divisions or groups of divisions of NACE, i.e. according to the A64 aggregations required by Eurostat (as the H_{PS}/H_{ZS} ratio is available according to these aggregations). Therefore, the deviation of the aggregated results for voivodships from the unknown true values will be significantly reduced. This is because voivodships differ in terms of the proportion between labour remuneration of working persons and labour remuneration of employed persons, mainly because of a different structure of NACE activities, which, to begin with, determines this differentiation. The above assumption, however, does not apply well enough in poorer economically developed countries where agriculture is a dominating element of the economy. For these countries, it is necessary to apply other methods of adjusting the remuneration of employed persons to the remuneration of working persons.[102]

In order to solve this problem in regional productivity accounting for the Polish economy (by voivodship), a hybrid solution was adopted, i.e. equation (63) was applied to all NACE sections except for section A (agriculture). For this one distinguished section the following equation was used:

$$WP_{PSW} = \left(WP_{ZS} + DM_S \frac{WP_{ZS}}{WDB_S - DM_S}\right)\frac{WDB_{SW}}{WDB_S}. \tag{64}$$

It is based on a concept assuming that a certain portion of the net operating surplus, including net mixed income DM_S at the level of the NACE section should be added to the labour remuneration for employed persons WP_{ZS} at the level of a given NACE section (agriculture, in this case). This surplus, together with the mixed income is assumed to be shared by labour and capital in the same proportion as the rest of the factors' income (i.e. WDB_S – DM_S) at the level of the NACE sections. These data come from TTs and have to be distributed by voivodships using the structure obtained from the BDL data repository on the basis of the proportions between the GVA for NACE sections at level of voivodships WDB_{SW} and the GVA for NACE sections at the WDB_S level of the entire Polish economy.[103]

At the end of this section, it is worth recalling that all increments can be calculated as the ratios of the subtractions of the current values for the previous year from the real values for the current year to the values of the previous year in current prices, or using logarithmic expressions. The calculation of values in constant base

[102] Equation (63) is a variant of one of the three ways of adjusting the labour share in factors' remuneration, i.e. in GVA, by the self-employment (see ILO, 2014, p. 173). This way has also been adopted in other productivity accountings (see OECD, 2001, pp. 39 and 45), which has its consequences (OECD, 2001, p. 47).

[103] This method is a developed variant of one of the three concepts presented by ILO (2014, p. 173). In other NACE sections sometimes also paid employees are not present, for example in section G (wholesale and retail trade), but basically only in section A methodological and accounting difficulties arise.

year prices (e.g. 2010) is not necessary, although possible and more convenient when using logarithmic expressions for the increments. If simple (not logarithmic) increments are used, a time series for relative real increments is obtained without the conversion procedure to the base year constant prices. The use of increments instead of levels – as already explained – generally reduces the deviations from the unknown true values. For positive values, the use of logarithms suppresses errors by reducing the outliers. The incremental values are usually positive, which can theoretically improve the results. However, for some NACE sections and at the level of provinces, and especially for NACE sections and at the level of provinces at the same time, i.e. generally at the level of lower aggregations, increments in the range of 10–20% or even over 20% occur more often. As a result, logarithmic approximation may produce more different results in relation to the ordinary denoted increments. At the same time, however, the use of logarithms instead of ordinary denoted increments contributes to the full compatibility of decomposition accounting with the production function (as discussed in Chapter 1). These problems are called tool problems because they are related to the limitations of mathematics that cannot always be overcome. However, performing the accountings according to both methods for the increments and obtaining similar results allows to confirm that the calculations have been performed correctly.

3.2. Performing Solow-type decomposition at regional level

The Solow-type[104] decomposition of economic growth, understood as the decomposition of the relative increase in GVA into inputs (contributions) of production factors (i.e. 'labour' and 'capital' factors) and the TFP contribution, for the aggregate of the whole economy and for lower aggregations – by NACE section (in A12 aggregations), voivodship and (which is important) simultaneously by NACE section and voivodship, was carried out for the Polish economy. The process was divided into two stages within grants obtained from two editions of the Technical Assistance Operational Programme (TAOP I and TAOP II).

During the first stage, the decomposition of the relative growth of GVA according to the A12 aggregations was performed in a simplified way, i.e. without determining the TFP contribution. For such a decomposition to be meaningful, it had to be performed into contributions of factor remuneration (labour and capital), i.e. according to the equation resulting from the transformation of equation (60). This simplified decomposition was performed for the contractually employed persons, not for working persons, following the methodology presented by Kotlewski (2015, 2017a).

[104] Solow (1957) proposed a decomposition of GDP growth, so we call the present decomposition of GVA growth a 'Solow-type' decomposition rather than 'Solow's' decomposition.

In the second stage, the decomposition of the relative growth of GVA, based on the classical idea of Solow (1957), was carried out. The decomposition was based on the general equation illustrated by (4), i.e. into contributions of factor stocks (resources), namely labour and capital resources, and into TFP contributions, according to a methodology described by Kotlewski (2018a, 2018b, 2019). At this stage, all accounts were performed both for the working persons and those (contractually) employed.

The methodology of the second stage, which will be presented in the further part of this work, contains the entire methodology of the performed regional productivity accounting (i.e. the methodology developed in the first stage). All the accounts were therefore performed in two ways: for the employed persons and the working persons. The equations used for both these categories are similar, differing only in the subscripts. However, since not all values are subscribed, as they are sometimes assumed to be common to both versions of the accounting, for clarity, the equations for employed persons and working persons are provided simultaneously. The difference between the equations for the decomposition of the relative GVA growth into factor remuneration contributions and the corresponding equations for the decomposition of the relative GVA growth into factor resources (stocks) contributions and TFP contributions is a different matter. Here, the difference between the respective equations is significant, because it arises at the level of the methodology of the accounting and not at the level of data preparation for the accounting, as in the case of the difference between the employed and working persons. In the cited equations, the original symbols used in the methodological sources have been retained to the greatest possible extent. They differ from the symbols used in KLEMS productivity accounting, but this also results from some discrepancies between the applied measures.

The scope of the study in the presented regional productivity accounting based on the factor decomposition of economic growth was extended not only by the spatial dimension. Each accounting was also performed *per capita*, which should be understood here as accounting per person participating in the production process, i.e. either per employed person or per working person. In addition, a decomposition of the deviations from the national average by all the aforementioned cross-sections and *per capita* was also carried out.

3.2.1. Macro- and mesoeconomic accounts

The decomposition of the relative growth of GVA into the contribution of labour remuneration and the contribution of capital remuneration was performed according to the equations below:

$$\Delta WDB/WDB_{(-1)} = \alpha_Z \Delta WP_Z / WP_{Z_{(-1)}} + \beta_Z \Delta WK_Z / WK_{Z_{(-1)}},$$
$$\Delta WDB/WDB_{(-1)} = \alpha_P \Delta WP_P / WP_{P_{(-1)}} + \beta_P \Delta WK_P / WK_{P_{(-1)}}, \tag{65}$$

where $\alpha_Z = (WP_Z/WDB + WP_{Z_{(-1)}}/WDB_{(-1)})/2$ and $\beta_Z = (WK_Z/WDB + WK_{Z_{(-1)}}/ /WDB_{(-1)})/2$ in a situation when the labour factor is understood as the remuneration of the (contractually) employed persons or $\alpha_P = (WP_P/WDB + WP_{P_{(-1)}}/ /WDB_{(-1)})/2$ and $\beta_P = (WK_P/WDB + WK_{P_{(-1)}}/WDB_{(-1)})/2$ in a situation when the labour factor is understood as the remuneration of the working persons. Subscript (–1) denotes – and will further denote – the value for the previous period, usually a yearly period.

If, according to the National Accounts, equation (60) is valid, then the relative (percentage) increase in GVA $\Delta WDB/WDB_{(-1)} = (WDB - WDB_{(-1)})/ WDB_{(-1)}$ is equal to the sum of relative (percentage) increases in labour remuneration $\Delta WP_Z/WP_{Z_{(-1)}} = (WP_Z - WP_{Z_{(-1)}})/ WP_{Z_{(-1)}}$ albo $\Delta WP_P/WP_{P_{(-1)}} = (WP_P - -WP_{P_{(-1)}})/ WP_{P_{(-1)}}$ and in capital remuneration $\Delta WK_Z/WK_{Z_{(-1)}} = (WK_Z - -WK_{Z_{(-1)}})/ WK_{Z_{(-1)}}$ or $\Delta WK_P/WK_{P_{(-1)}} = (WK_P - WK_{P_{(-1)}})/ WK_{P_{(-1)}}$ for the employed persons or working persons respectively, weighted (multiplied) by the shares of these factors in GVA. This is strictly the case if the increments are infinitesimally small, i.e. in continuous time.

If time is not treated as continuous but as discrete, i.e. when there are measurable time intervals, then the α and β weights (with appropriate subscripts for either employed persons or working persons) should be used in the form of average intertemporal factor shares in GVA according to the formulae given above (so a linear interpolation of shares is performed between current and previous periods). This means that in discrete time equations (65) are subject to some small deviations from the unknown true values, so they are approximate equations, as linear interpolation is an approximate procedure. In order for this small deviation not to increase in further calculations, for the capital remuneration contribution (WWK_Z or WWK_P for employed or working persons, respectively), instead of $\beta_Z \Delta WK_Z / WK_{Z_{(-1)}}$ or $\beta_P \Delta WK_P / WK_{P_{(-1)}}$ the values calculated residually according to the following equations are applied:

$$WWK_Z = \Delta WDB/WDB_{(-1)} - \alpha_Z \Delta WP_Z / WP_{Z_{(-1)}},$$
$$WWK_P = \Delta WDB/WDB_{(-1)} - \alpha_P \Delta WP_P / WP_{P_{(-1)}}. \tag{66}$$

The contribution of the remuneration of capital to the relative growth of GVA is thus calculated by subtracting the contribution of the remuneration of labour from the growth of GVA. A similar procedure is continued with other capital contributions, which ensures formal accuracy and the balancing of the account.

In turn, the decomposition of the relative GVA growth into contributions of the labour stock (resource), capital stock (resource) and TFP was performed according to equations:

$$\Delta WDB/WDB_{(-1)} = \frac{\alpha_z \Delta Z}{Z_{(-1)}} + \frac{\beta_z \Delta K}{K_{(-1)}} + \frac{\Delta TFP_Z}{TFP_{Z(-1)}},$$
$$\Delta WDB/WDB_{(-1)} = \frac{\alpha_p \Delta P}{P_{(-1)}} + \frac{\beta_p \Delta K}{K_{(-1)}} + \frac{\Delta TFP_P}{TFP_{P(-1)}}. \tag{67}$$

These equations demonstrate that the variable representing the value of capital stock K (i.e. the stock of fixed capital) does not vary with the variable associated with the 'labour' factor, i.e. for the number of employed persons Z or for the number of working persons P, and is therefore not subscribed. But the share of capital remuneration in GVA β changes and takes the values of β_z or β_p, respectively. Equations (65) are exempt from the need to determine β_z or β_p when adopting the calculations provided in equations (66); here, however, it is necessary to determine β_z or β_p in order to make equations (67) solvable. This accounting is therefore more demanding from the point of view of the input data necessary for the decomposition calculus. To avoid this problem, which can sometimes be difficult to solve, one can assume that $\beta = 1 - \alpha$ (with appropriate Z or P subscripts), based on the nearly always adopted assumption of constant returns to scale in an economy operating under perfect competition.

The shares of factors α and β (with the respective Z or P subscripts) in equations (67) are basically the same as in equations (65), except that in the latter there was no need to determine β_z or β_p. All the other variables on the right-hand side of the equations must always be calculated either for the number of employed persons Z only, or for the number of working persons P only. For the sake of accuracy, it is necessary to present here how the increment of the labour factor is calculated for the number of employed persons: $\Delta Z/Z_{(-1)} = (Z - Z_{(-1)})/\, Z_{(-1)}$ or for the number of working persons: $\Delta P/P_{(-1)} = (P - P_{(-1)})/\, P_{(-1)}$ and how the increment of the capital factor is calculated: $\Delta K/K_{(-1)} = (K - K_{(-1)})/\, K_{(-1)}$ (for both employed and working persons).

Theoretically, the increment in TFP should also be calculated in this way: $\Delta TFP_Z/TFP_{Z(-1)} = (TFP_Z - TFP_{Z(-1)})/\, TFP_{Z(-1)}$ or $\Delta TFP_P/TFP_{P(-1)} = (TFP_P - TFP_{P(-1)})/$ $/\, TFP_{P(-1)}$ for either employed or working persons, respectively. This variable represents, according to the theory, the contribution of technological and organisational progress to economic growth (with the possibility of considering the interpretation variant for this variable, which is applied to the decomposition of the relative growth of GVA, according to which TFP is interpreted as the ability to capture value), i.e. the TFP contribution ($WTFP_Z$ or $WTFP_P$, respectively). This variable, however, tends to be calculated rather residually, similarly to the capital variable in equations (66), i.e. according to equations:

$$
\begin{aligned}
WTFP_Z &= \frac{\Delta WDB}{WDB_{(-1)}} - \alpha_z \frac{\Delta Z}{Z_{(-1)}} - \beta_z \frac{\Delta K}{K_{(-1)}}, \\
WTFP_P &= \frac{\Delta WDB}{WDB_{(-1)}} - \alpha_p \frac{\Delta P}{P_{(-1)}} - \beta_p \frac{\Delta K}{K_{(-1)}}.
\end{aligned}
\tag{68}
$$

However, here it is not an alternative action, as in the case of equations (66), but a necessary one, because there is no other way to determine the contributions of TFP as this variable is not directly observable.

In order for equations (68) to be solvable, it is also necessary to determine β_z or β_p. The adoption of the basic theoretical assumption of constant returns to scale under perfect competition, namely that $\beta = 1 - \alpha$ (with appropriate Z or P indices), greatly facilitates this task and additionally solves the technical problem of a minor tool inconsistency resulting from the calculation of both parameters by linear interpolation, similarly to what was indicated in the comments to equations (65).

In all the equations above, the subscripts that relate to the different aggregations at the level of which all these formulae were applied have been omitted, i.e. for the aggregate of the whole economy, for different NACE sections (or groups of NACE sections), different voivodships and simultaneously for various NACE sections and different voivodships.

The fact that two classifications are used: PKD 2004 (Polish equivalent of NACE rev. 1.1) and PKD 2007 (Polish equivalent of NACE rev. 2), creates some difficulties at the interface between these classifications with regard to the calculation of growth rates, including relative growth rates. Data for 2008 are available in both classifications, therefore increments ('deltas') between 2007 and 2008 should be calculated on the basis of data according to the PKD 2004 classification, and increments between 2008 and 2009 according to the PKD 2007 classification. Thus, data for 2008 should be used according to either of the classifications, depending on the situation. This practice almost completely levels out, as observation shows, the effects of inconsistencies resulting from the use of different classification systems.

3.2.2. Production factor remuneration accounts *per capita*

A special feature of all types of the discussed accounting is that they were also performed per person involved in the production process, i.e. per employed person (Z) or working person (P). This is also sometimes done by some countries implementing KLEMS productivity accounting. In particular, it concerns those countries that implement a full GVA decomposition on the EU KLEMS platform, for which all decomposition elements are presented for the aggregate of the whole economy and for sectoral sub-aggregations (the 34 lowest EU KLEMS aggregations, i.e. the A34 aggregation), as well as per hour worked and per working person. A decomposition by region of a given country is not made (with the exception of Spain – the website of the statistical office of this country publishes information related to the decomposition accounting performed at the provincial level, but this regional decomposition is not available on the EU KLEMS website platform).

The accounting presented in this work was also performed at the section and region level (by NACE section, by voivodship and by NACE section and voivodship simultaneously), although on sectoral aggregations higher than the EU KLEMS system (A12 aggregations instead of A34 aggregations). It was not possible to do them per hour worked, but they were done for both the (contractually) employed persons and the working persons.

For increments per employed or working person, equations (65) should be changed to the form presented below:

$$\frac{\Delta(WDB/Z)}{WDB_{(-1)}/Z_{(-1)}} = \alpha_Z \frac{\Delta((WP_Z)/Z)}{WP_{Z(-1)}/Z_{(-1)}} + \beta_Z \frac{\Delta((WK_Z)/Z)}{WK_{Z(-1)}/Z_{(-1)}},$$
$$\frac{\Delta(WDB/P)}{WDB_{(-1)}/P_{(-1)}} = \alpha_P \frac{\Delta((WP_P)/P)}{WP_{P(-1)}/P_{(-1)}} + \beta_P \frac{\Delta((WK_P)/P)}{WK_{P(-1)}/P_{(-1)}}, \tag{69}$$

where Z is the number of employed persons in the current period, $Z_{(-1)}$ is the number of employed persons in the previous period, P is the number of working persons in the current period, and $P_{(-1)}$ is the number of working persons in the previous period. In equations (69), the following formulae apply: $\Delta(WDB/Z) = WDB/Z - WDB_{(-1)}/Z_{(-1)}$, $\Delta(WP_Z/Z) = WP_Z/Z - WP_{Z(-1)}/Z_{(-1)}$ and $(WK_Z/Z) = WK_Z/Z - WK_{Z(-1)}/Z_{(-1)}$, and the formulae $\Delta(WDB/P) = WDB/P - WDB_{(-1)}/P_{(-1)}$, $\Delta(WP_P/P) = WP_P/P - WP_{P(-1)}/P_{(-1)}$ and $\Delta(WK_P/P) = WK_P/P - WK_{P(-1)}/P_{(-1)}$. In practice, however, the contribution of capital remuneration to GVA growth per employed persons or per working persons ($WWKZ$ or $WWKP$ – slightly different symbols from those in equations (66)) is not determined from the expressions $\beta_Z\Delta(WK_Z/Z)/(WK_{Z(-1)}/Z_{(-1)})$ and $\beta_P\Delta(WK_P/P)/(WK_{P(-1)}/P_{(-1)}$, but by calculating it residually according to equations:

$$WWKZ = \frac{\Delta(WDB/Z)}{WDB_{(-1)}/Z_{(-1)}} - \alpha_z \frac{\Delta((WP_Z)/Z)}{WP_{Z(-1)}/Z_{(-1)}},$$
$$WWKP = \frac{\Delta(WDB/P)}{WDB_{(-1)}/P_{(-1)}} - \alpha_p \frac{\Delta((WP_P)/P)}{WP_{P(-1)}/P_{(-1)}}, \qquad (70)$$

i.e. by subtracting the contributions of the labour factor for the employed persons or the working persons from the relative growth of GVA per employed person or per working person.

In turn, the deviations of GVA per employed or working person for a given voivodship, for a given section of NACE or for a given voivodship and a given section of NACE simultaneously (or possibly other selected aggregations) in relation to the national average and the contributions of factors to these deviations should theoretically fulfil the following equations:

$$\frac{WDB_j/Z_j - WDB/Z}{WDB/Z} = \alpha_{zj} \frac{WP_{Zj}/Z_j - WP_Z/Z}{WP_Z/Z} + \beta_{zj} \frac{WK_{Zj}/Z_j - WK_Z/Z}{WK_Z/Z},$$
$$\frac{WDB_j/P_j - WDB/P}{WDB/P} = \alpha_{pj} \frac{WP_{Pj}/P_j - WP_P/P}{WP_P/P} + \beta_{pj} \frac{WK_{Pj}/P_j - WK_P/P}{WK_P/P}. \qquad (71)$$

Here, subscript j indicates the values for a given voivodship, a given NACE section or a given voivodship and a given NACE section simultaneously (or possibly other selected aggregations), while the values not indexed by j refer to the entire country. In this case, the use of index j is necessary to ensure the readability of the equations, as opposed to the previous equations, for which it was omitted for clarity.

For the same reason as for the previous equations, the contributions of the remuneration of capital to the deviations of GVA per employed person ($WWKO_Z$) or working person ($WWKO_P$) are calculated in a residual manner from the equations below:

$$WWKO_Z = \frac{WDB_j/Z_j - WDB/Z}{WDB/Z} - \alpha_z \frac{WP_{Zj}/Z_j - WP_Z/Z}{WP_Z/Z},$$
$$WWKO_P = \frac{WDB_j/P_j - WDB/P}{WDB/P} - \alpha_p \frac{WP_{Pj}/P_j - WP_P/P}{WP_P/P}, \qquad (72)$$

instead of using the expressions $\beta_Z(WK_{Zj}/Z_j–WK_Z/Z)/(WK_Z/Z)$ or $\beta_p(WK_{Pj}/Z_j–$ $–WK_P/Z)/(WK_P/Z)$. Weights α_z and β_Z are calculated in a different way here, i.e. from the following formulae: $\alpha_Z = WP_Z/WDB$ oraz $\beta_Z = WK_Z/WDB$, and the weights α_p and β_p – from the formulae: $\alpha_p = WP_P/WDB$ and $\beta_p = WK_P/WDB$. Therefore, they are not calculated by linear interpolation between two periods, as in other cases, because the data used in equations (71) and (72) are always from one period only. For the sake of the readability of the equations, additional indices have been omitted.

In general, the weights used throughout the accounting are the arithmetic averages of the weights from two periods, i.e. the previous and the current one, at each level of aggregation, following the Törnqvist procedure for comparing two periods or two situations. However, it is different in the case of deviations. Data do not come from two periods, but from two situations (different voivodships and different sections of PKD) at the same time. Therefore, theoretically, the arithmetic averages from two situations at the same time should be used. However, it would be incorrect, since we are comparing two units from different taxonomic levels, i.e. voivodships with the entire Polish economy. That is why weights for the whole country are used for the deviations, treating the weights as a relevant point of reference, while retaining their differentiation according to NACE sections as in the whole accounting (which is not, however, distinguished here by appropriate indices in order to maintain the readability of the equations).

In addition to the accounting above, a decomposition of the changes in the deviation from the national average was performed. This accounting allows a clearer observation of whether the difference from the national average is increasing or decreasing. It requires the addition of the Δ symbol in the relevant places in equations (71) and (72). For clarity, these equations, appropriately transformed, are provided below:

$$
\begin{gathered}
\Delta\frac{WDB_j/Z_j - WDB/Z}{WDB/Z} = \alpha_z\Delta\frac{WP_{Zj}/Z_j - WP_Z/Z}{WP_Z/Z} + \beta_z\Delta\frac{WK_{Zj}/Z_j - WK_Z/Z}{WK_Z/Z}, \\
\Delta\frac{WDB_j/P_j - WDB/P}{WDB/P} = \alpha_p\Delta\frac{WP_{Pj}/P_j - WP_P/P}{WP_P/P} + \beta_p\Delta\frac{WK_{Pj}/P_j - WK_P/P}{WK_P/P}, \\
\Delta WWKO_Z = \Delta\frac{WDB_j/Z_j - WDB/Z}{WDB/Z} - \alpha_z\Delta\frac{WP_{Zj}/Z_j - WP_Z/Z}{WP_Z/Z}, \\
\Delta WWKO_P = \Delta\frac{WDB_j/P_j - WDB/P}{WDB/P} - \alpha_p\Delta\frac{WP_{Pj}/P_j - WP_P/P}{WP_P/P}.
\end{gathered}
\tag{73}
$$

In the above, $\Delta WWKO_Z$ is the contribution of the remuneration of capital to the change in the deviation of GVA per employed person and $\Delta WWKO_P$ is the same contribution but per working person.

In all the equations above, as in paragraph 3.2.1, subscripts related to the fact that calculations were made using these equations for the aggregate of the whole economy, for sections of NACE, for voivodships, and for sections of NACE and voivodships simultaneously, with the exception of the above-mentioned deviations (to which reference is made above), have been omitted. Subscript t, associated with the periods for which all of the equations above are used, has also been omitted, lest they become completely unreadable. The comment made in paragraph 3.2.1 on how to calculate increments when using two classification systems, PKD 2004 (Polish equivalent of NACE rev. 1.1.) and PKD 2007 (Polish equivalent of NACE rev. 2), should also be considered here.[105]

3.2.3. *Per capita* accounts with TFP extraction

The simplified decomposition of the relative growth of GVA into factor remuneration contributions made per person involved in the production process, represented by equations (69), can be developed into a full Solow-inspired decomposition according to general equation (4). For this purpose, equations (67) for the aggregate of the whole economy should be appropriately transformed into the following:

$$\begin{aligned}
\frac{\Delta(WDB/Z)}{WDB_{(-1)}/Z_{(-1)}} &= \alpha_z \frac{\Delta(Z/Z)}{Z_{(-1)}/Z_{(-1)}} + \beta_z \frac{\Delta(K/Z)}{K_{(-1)}/Z_{(-1)}} + \frac{\Delta(TFP_Z/Z)}{TFP_{Z(-1)}/Z_{(-1)}}, \\
\frac{\Delta(WDB/P)}{WDB_{(-1)}/P_{(-1)}} &= \alpha_p \frac{\Delta(P/P)}{P_{(-1)}/P_{(-1)}} + \beta_p \frac{\Delta(K/P)}{K_{(-1)}/P_{(-1)}} + \frac{\Delta(TFP_P/P)}{TFP_{P(-1)}/P_{(-1)}}.
\end{aligned} \tag{74}$$

The previous formulae are accompanied by explanations of all the symbols used in the equations above. There is a peculiarity in equations (74) – the values associated with the labour factor can be truncated. Therefore, these equations simplify to the following forms:

$$\begin{aligned}
\frac{\Delta(WDB/Z)}{WDB_{(-1)}/Z_{(-1)}} &= \beta_z \frac{\Delta(K/Z)}{K_{(-1)}/Z_{(-1)}} + \frac{\Delta(TFP_Z/Z)}{TFP_{Z(-1)}/Z_{(-1)}}, \\
\frac{\Delta(WDB/P)}{WDB_{(-1)}/P_{(-1)}} &= \beta_p \frac{\Delta(K/P)}{K_{(-1)}/P_{(-1)}} + \frac{\Delta(TFP_P/P)}{TFP_{P(-1)}/P_{(-1)}}.
\end{aligned} \tag{75}$$

Equations (75) do not contain parameters α_z and α_p, but their magnitudes are available, and the assumption of constant returns to scale under perfect competition allows here also the application of equation $\beta = 1 - \alpha$ (with appropriate indices Z or P) in order to determine parameter β. This, in turn, makes it possible to determine the contribution of TFP per employed person $\left(WTFP_{\Delta V/Z}\right)$ or per working person $\left(WTFP_{\Delta V/P}\right)$:

[105] We refer to Polish equivalents of the two NACE systems here because the difference between them is not exactly the same as between the two NACE systems. We will further denote them NACE 1 and NACE 2.

$$WTFP_{\Delta V/Z} = \frac{\Delta(WDB/Z)}{WDB_{(-1)}/Z_{(-1)}} - \beta_z \frac{\Delta(K/Z)}{K_{(-1)}/Z_{(-1)}},$$
$$WTFP_{\Delta V/P} = \frac{\Delta(WDB/P)}{WDB_{(-1)}/P_{(-1)}} - \beta_p \frac{\Delta(K/P)}{K_{(-1)}/P_{(-1)}}. \qquad (76)$$

Calculations including the determination of TFP for the deviations from the national average GVA per employed person or per working person were also performed. Equations (71) should in this case be replaced by the equations below:

$$\frac{WDB_j/Z_j - WDB/Z}{WDB/Z} = \alpha_z \frac{Z_j/Z_j - Z/Z}{Z/Z} + \beta_z \frac{K_j/Z_j - K/Z}{K/Z} + \frac{TFP_{Zj}/Z_j - TFP_Z/Z}{TFP_Z/Z},$$
$$\frac{WDB_j/P_j - WDB/P}{WDB/P} = \alpha_p \frac{P_j/P_j - P/P}{P/P} + \beta_p \frac{K_j/P_j - K/P}{K/P} + \frac{TFP_{Pj}/P_j - TFP_P/P}{TFP_P/P} \qquad (77)$$

These equations also have the aforementioned peculiarity related to the labour factor, which allows their simplification to the form of:

$$\frac{WDB_j/Z_j - WDB/Z}{WDB/Z} = \beta_z \frac{K_j/Z_j - K/Z}{K/Z} + \frac{TFP_{Zj}/Z_j - TFP_Z/Z}{TFP_Z/Z},$$
$$\frac{WDB_j/P_j - WDB/P}{WDB/P} = \beta_p \frac{K_j/P_j - K/P}{K/P} + \frac{TFP_{Pj}/P_j - TFP_P/P}{TFP_P/P}. \qquad (78)$$

Here again, the assumption of constant returns to scale under perfect competition allows the use of formula $\beta = 1 - \alpha$ (with corresponding indices Z or P). This, in turn, makes it possible to determine the contribution of TFP to the deviation of GVA per employed person ($WTFPO_Z$) or per working person ($WTFPO_P$) by necessity in a residual manner:

$$WTFPO_Z = \frac{WDB_j/Z_j - WDB/Z}{WDB/Z} - \beta_z \frac{K_j/Z_j - K/Z}{K/Z},$$
$$WTFPO_P = \frac{WDB_j/P_j - WDB/P}{WDB/P} - \beta_p \frac{K_j/P_j - K/P}{K/P}. \qquad (79)$$

As a complement to the accounting above, decompositions of the changes in the deviations from the national averages were also performed. The accounting requires the addition of the Δ symbol in equations (78) and (79) in the necessary places. For clarity, these equations, appropriately transformed, are provided below:

$$\Delta \frac{WDB_j/Z_j - WDB/Z}{WDB/Z} = \beta_z \Delta \frac{K_j/Z_j - K/Z}{K/Z} + \Delta \frac{TFP_{Zj}/Z_j - TFP_Z/Z}{TFP_Z/Z},$$

$$\Delta\frac{WDB_j/P_j-WDB/P}{WDB/P}=\beta_p\Delta\frac{K_j/P_j-K/P}{K/P}+\Delta\frac{TFP_{Pj}/P_j-TFP_P/P}{TFP_P/P},$$

$$\Delta WTFPO_Z=\Delta\frac{WDB_j/Z_j-WDB/Z}{WDB/Z}-\beta_z\Delta\frac{K_j/Z_j-K/Z}{K/Z}, \tag{80}$$

$$\Delta WTFPO_P=\Delta\frac{WDB_j/P_j-WDB/P}{WDB/P}-\beta_p\Delta\frac{K_j/P_j-K/P}{K/P}.$$

In all the equations above, as in paragraphs 3.2.1 and 3.2.2, the subscripts relating to aggregations have been omitted. Only in the case of the deviations from the averages has subscript j been used for the value compared to the national average, in order to make these equations readable in terms of content. For clarity, subscript t, relating to the periods for which all the above equations are used, has been omitted everywhere. It is also advisable to bear in mind the calculation procedure of the increments in a situation involving the simultaneous use of the PKD 2004 (NACE 1) and PKD 2007 (NACE 2) classifications.

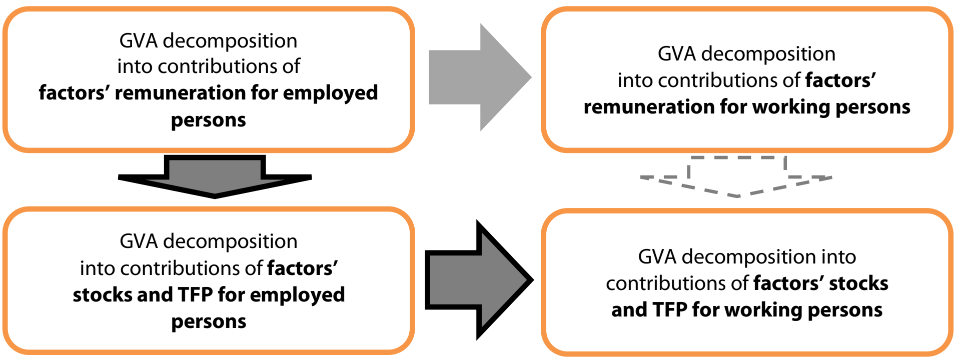
3.2.4. Two ways of performing the calculations

The presented calculations could be performed in two ways. The first stage (Kotlewski, 2015, 2017a) consisted in performing the decomposition of the relative growth of GVA into the contributions of factor (labour and capital) remuneration only and only for the employed persons. The second stage (Kotlewski, 2018a, 2018b, 2018c, 2019) involved the transformation of the accounting into a decomposition of the relative growth of GVA into the contributions of factor (labour and capital) resources (stocks) and the contribution of TFP for the employed persons. This required the replacement of some data by other new data, but at the same time the set of computational algorithms from the first stage was partly used together with some data that were retained or only transformed. Thus, the initial calculus of the decomposition of the relative growth of GVA into factor remuneration contributions underwent a kind of conversion into a calculus of a decomposition into factor resources (stocks) contributions and a TFP contribution. This process is illustrated by the left vertical arrow in Figure 9.

The other transformations of the accounting were dealt with in a similar way, i.e. by a kind of conversion consisting in substituting, transforming and possibly preserving data by means of the existing, slightly modified set of algorithms. The converted decomposition accounting (which was already an accounting of a classical Solow type decomposition (into factor resources (stocks) contributions and TFP contribution), but for the employed persons) was transformed into an accounting for the working persons. This is illustrated by the lower horizontal arrow in Figure 9.

In this way, a full Solow-type decomposition for working persons was achieved, which is the most appropriate of the four presented from the point of view of the prevailing decomposition in the theory of economic growth and which inspired the decomposition within KLEMS productivity accounting.

Figure 9. Sequence of calculations in the Solow-type regional decomposition in the actually performed accountings



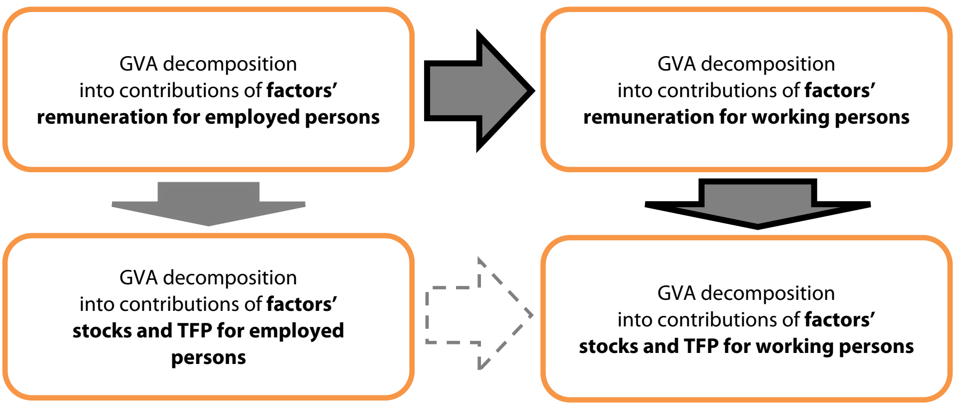
Source: author's work.

In addition, in order to obtain the full possible results in all four variants, the accounting of the decomposition of the relative growth of GVA into remuneration contributions of the mentioned factors for the employed persons was converted to such a decomposition accounting for the working persons, as shown by the upper horizontal arrow in Figure 9. Having the full accounting performed according to the designed research, the still possible operation, marked by the right vertical arrow in Figure 9, was not performed.

However, reaching this last stage is also possible in another way, as shown in Figure 10, first by performing the conversion of the relative growth decomposition accounting of GVA into factor (labour and capital) remuneration contributions for the employed persons into such accounting for the working persons (as shown by the upper horizontal arrow in Figure 10), and then converting the latter accounting into relative growth decomposition accounting of GVA into factor resources (stocks) contributions and TFP contribution for the working persons, as shown by the right vertical arrow in Figure 10.

If the second way was used, the accounting would have to be complemented by the conversion of the decomposition of the relative growth of GVA into factor (labour and capital) remuneration contributions for the employed persons (realised at the beginning of this work) into accounting of the decomposition of the relative growth of GVA into factor (labour and capital) resources (stocks) and TFP contributions for the employed persons, as indicated by the left vertical arrow in Figure 10. In this situation, the conversion of the accounting indicated by the lower horizontal arrow in Figure 10 would be redundant.

Figure 10. Sequence of calculations in the Solow-type regional decomposition in the optional accounting



Source: author's work.

However, the accounting presented in Figure 10 was not carried out because the results would have been identical or differed only in terms of tool-alike deviations and only very slightly from the results actually obtained, i.e. to an extent irrelevant from the point of view of economic analysis.[106]

3.2.5. Analytical benefits of performing decomposition in four variants

The advantage of the described factor decompositions complemented by a full Solow-type decomposition[107] over KLEMS productivity accounting carried out in Poland so far is their spatial dimension. While KLEMS productivity accounting was carried out only for the aggregate country's economy, the simpler factor

[106] In light of the further discussion concerning the analytical benefits of performing the decomposition in four variants, perhaps such an approach would be more appropriate. However, the small accounting benefits of tool-alike provenance do not justify undertaking such a large amount of work.

[107] Basically, the accounting differs from Solow's decomposition only in the fact that the decomposed value of the relative growth is GVA, not GDP.

decomposition accounting was also performed by individual voivodships (moreover, they were also carried out per person engaged in the production process and for the deviations from the average, as shown in sections 3.2.1–3.2.3, but this is relatively easy to do within the framework of the performed KLEMS accounting[108]). Therefore, the analytical value of these types of decomposition accounting, including the Solow-type decompositions, is that they allow the factor analysis of economic growth to include the spatial dimension. In this respect, it turns out that performing the decomposition in four variants is analytically valuable.

It should be mentioned here that performing accounting for the employed persons has a certain value in itself (see Kotlewski, 2015, 2017a), since it analyses and compares the remuneration contribution of the employed persons to the remuneration contribution of the owners of the means of production, that is, in the latter case, to the remuneration contribution of capital together with the remuneration of the self-employed for their own work – this is particularly often the case for small business owners whose remuneration for merely owning the means of production is indistinguishable from the part of their remuneration that they pay themselves for their own work.

However, in the light of the theory, in principle, factor analysis favours a reasonably strict separation of the labour factor from the capital factor. The conversion of the accounting made for the employed persons into accounting for the working persons is then essential (even though the additional conversions introduce additional biases into the accounting, including tool-like ones). This is especially true for the decomposition into factor resources (stocks) contributions and a TFP contribution.

When the contributions of the resource (stock) of working persons and of the stock (resource) of capital and of TFP to the relative growth of GVA are taken into account, the interpretation of the growth of factors and their contributions does not pose problems. However, the case is different for the contributions of the resource (stock) of employed persons and the stock (resource) of capital and TFP. This is due to the fact that in the latter case the remuneration of the self-employed increases the share of capital in the GVA; as a result β_Z is larger than β_P. The contribution of the capital stock to the relative growth of GVA is therefore overestimated, which is difficult to interpret analytically. However, this overestimation of the contribution of the capital stock compensates for the reduction in the contribution of the labour stock (resource), resulting from the underestimation by the self-employment of the value

[108] Some countries performing KLEMS accounting, including on the EU KLEMS platform, publish these data, sometimes also per hour worked and not only per working person. There is no significant methodological problem in performing the additional calculations involved.

for the employed persons to the value for the working persons. It follows that the residual TFP contribution is similar for both versions of the decomposition, but this compensation is unfortunately not completely symmetric. On the other hand, if the same β-share as for the working persons, i.e. β_P, were used, then the residual TFP contribution would be overestimated. As a result, performing a decomposition accounting with a TFP extraction de facto means that it has to be converted into accounting for the working persons. The analyses presented in Figures 11 and 12 therefore apply to comparisons without the decomposition in the version represented by the lower left rounded rectangle in Figures 9 and 10.

For Figures 11 and 12, compound growth rates were analysed and calculated for six-year periods. For GVA this was done according to the equation below:

$$\Delta V_{t_{1-6}} = \left(1 + \Delta V_{t_1}\right)\left(1 + \Delta V_{t_2}\right)\left(1 + \Delta V_{t_3}\right)\left(1 + \Delta V_{t_4}\right)\left(1 + \Delta V_{t_5}\right) \\ \left(1 + \Delta V_{t_6}\right) - 1, \tag{81}$$

where $\Delta V_{t_{1-6}}$ denotes the compound relative growth of GVA over the given six-year period, and ΔV_{t_0}, ΔV_{t_1}, … ΔV_{t_6} – the successive one-year relative growth rates in successive years of the period, denoted by the numbers 1, 2, ... 6.

For compound factor contributions, the following equation was used:

$$\gamma \Delta F_{t_{1-6}} = \left(1 + \gamma_1 \Delta F_{t_1}\right)\left(1 + \gamma_2 \Delta F_{t_2}\right)\left(1 + \gamma_3 \Delta F_{t_3}\right) \\ \left(1 + \gamma_4 \Delta F_{t_4}\right)\left(1 + \gamma_5 \Delta F_{t_5}\right)\left(1 + \gamma_6 \Delta F_{t_6}\right) - 1, \tag{82}$$

where $\gamma \Delta F_{t_{1-6}}$ is the compound contribution of a factor (F – *factor*), e.g. labour or capital, and ΔF_{t_1}, ΔF_{t_2}, … ΔF_{t_6} the successive annual growth rates of the given factor, weighted by its value shares γ_1, γ_2, … γ_6 in the GVA in the successive years of the six-year period.

As regards the compound contribution of TFP to the relative growth of GVA, it was calculated residually as the difference between the compound relative growth rates of GVA and the sum of the compound factor (labour and capital) contributions at each level of aggregation. This procedure was chosen so that the tool bias would not build up during the calculations.

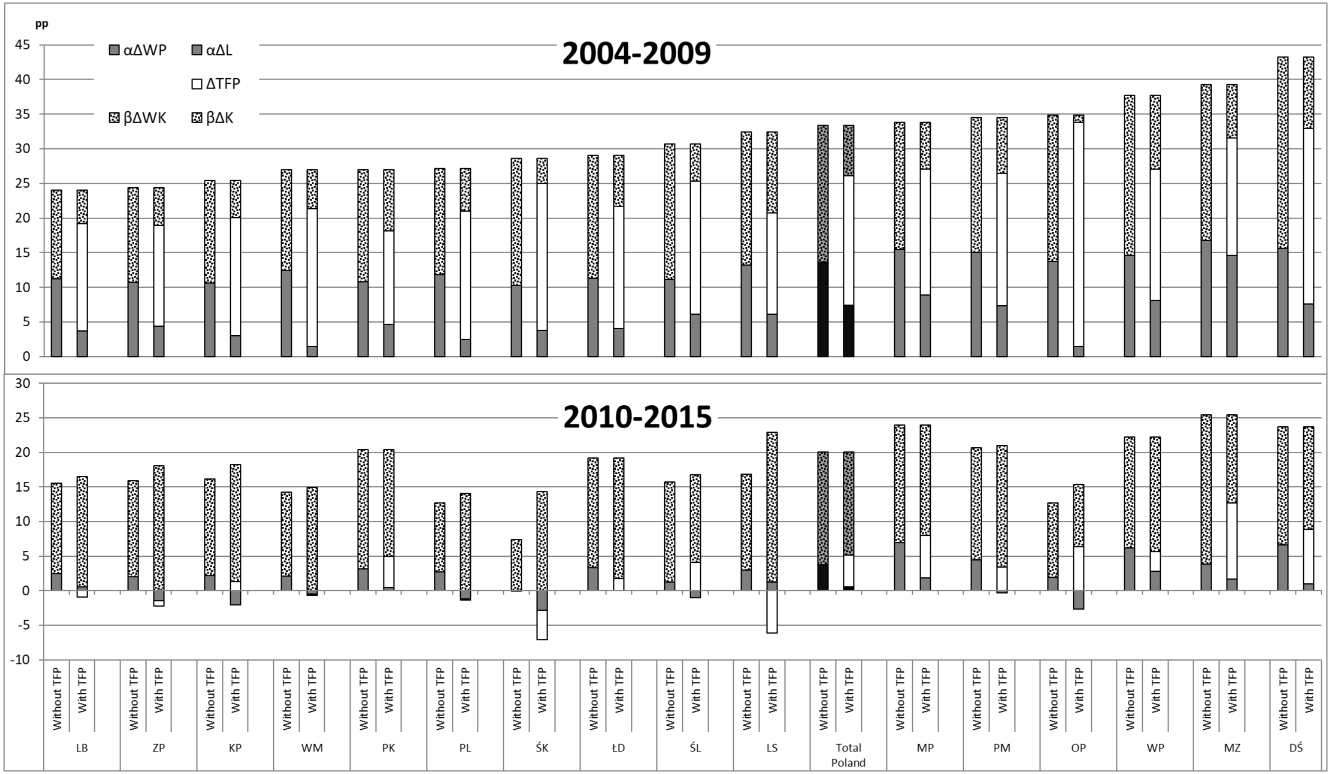
Data from these calculations are presented on Figures 11 and 12. Figure 11 was prepared for the employed persons. There are two bars for each voivodship and the whole Polish economy – the left one for the decomposition of the relative growth of GVA into factor remuneration, and the right one for the decomposition of the relative growth of GVA into factor resources (stocks) contributions and a TFP contribution.

In Figure 11 (the upper graph, for the period 2004–2009), voivodships are ranked in order from the lowest to the highest compound relative GVA growth rates. The compound relative growth rate of GVA for the aggregate Polish economy has also been added as reference. On the right-hand side from the value for 'Total Poland' are voivodships developing faster in relative terms than the aggregate Polish economy in the years 2004–2009. The opposite is true for voivodships located on the left-hand side from the value for the aggregate Polish economy. Due to the limited readability of the graphs, it should be noted that the sub-bars lying right next to the horizontal axis refer to labour factor contributions, the sub-bars at the top of the bars to capital factor contributions, and the white sub-bars to TFP contributions.

In Figure 11, in the lower graph (concerning the period 2010–2015), identical assumptions were made and the order of the ranking of voivodships was retained. It was still possible to extend the results by 2003, however, the idea was abandoned in order to ensure that a full comparability for equal six-year periods is retained. In both graphs (upper and lower), two bars refer to each voivodship and the aggregate country's economy. The left-hand bar illustrates the decomposition represented by the right-hand upper rectangle in Figures 9 and 10, and the one on the right the decomposition represented by the right lower rectangle in these figures, i.e. these are graphs for the working persons. The right-hand bars of Figure 11 for the period of 2010–2015 show that the contribution of the TFP to the relative growth of GVA was the highest among all the distinguished factor contributions in the period 2004–2009. In the case of some voivodships, e.g. Dolnośląskie (DŚ in Figure 11 for the period of 2010–2015), it was even higher than all the other distinguished contributions combined; the same case was for the aggregate national economy in the period of 2004–2009.

Performing the decomposition of the relative growth of GVA also into factor remuneration contributions (left-hand bars on Figure 11) makes it possible to determine what part of the TFP contribution is being transformed into a part of the labour remuneration contribution, and what part into the capital remuneration contribution. It can be concluded that in the period 2004–2009 the TFP contribution resulted in a greater increase in the capital remuneration contribution than in the labour remuneration contribution for most voivodships and for the entire country (except for e.g. the Warmińsko-Mazurskie voivodship). This means that in general in that period the productivity of capital grew faster than the productivity of labour (assuming that factors are remunerated according to their marginal productivities). The decomposition by voivodship allows the observation of the spatial distribution of the impact of the TFP on the growth of factors remuneration.

Figure 11. Comparison of the decomposition of compound relative growth of GVA with and without a TFP extraction



Note. Contributions of: $\alpha \Delta WP$ – labour remuneration, $\beta \Delta WK$ – capital remuneration, $\alpha \Delta L$ – labour resource, $\beta \Delta K$ – capital stock, ΔTFP – TFP. Voivodships: LB – Lubelskie, ZP – Zachodniopomorskie, KP – Kujawsko-Pomorskie, WM – Warmińsko-Mazurskie, PK – Podkarpackie, PL – Podlaskie, ŚK – Świętokrzyskie, ŁD – Łódzkie, ŚL – Śląskie, LS – Lubuskie, MP – Małopolskie, OP – Opolskie, WP – Wielkopolskie, MZ – Mazowieckie, DŚ – Dolnośląskie.

Source: Kotlewski (2019).

What turns out to be even more interesting is the observation that the Polish economy underwent significant changes in the period of 2010–2015, including a slight slowdown. The compound relative growth of GVA in the period of 2004–2009 reached approximately 30%, while in the period of 2010–2015 about 20%. The order of the fastest growing voivodships changed. Dolnośląskie Voivodship was no longer the leader of economic growth, understood as the relative growth of GVA, but Mazowieckie Voivodship. Other changes in the ranking between voivodships also occurred in terms of the pace of the above-mentioned growth. The contribution of TFP at the level of the aggregate national economy ceased to be the most important contribution to the relative growth of GVA (from among the distinguished contributions in the decomposition accounting which took into consideration the extraction of the TFP). The decrease in the importance of the TFP contribution also resulted in cases of a negative TFP contribution in the situation of its differentiation between voivodships. Moreover, while in the period 2004–2009 most of the TFP contribution to relative GVA growth was transformed into the contribution of capital remuneration, in the period of 2010–2015 this productivity contribution was mainly associated with the corresponding part of the contribution of labour remuneration. Considering the aforementioned theoretical assumptions, this means that labour productivity started to grow faster than capital productivity. However, this was not the case for some voivodships, including, interestingly, Mazowieckie Voivodship, where capital productivity continued to grow faster than labour productivity in the 2010–2015 period.

In general, this phenomenon is consistent with the global trend of declining capital productivity growth observed by Acemoglu (2003) and Klump et al. (2004). This decline, however, was accompanied both in Poland and globally by an increase in the share of capital contribution and the contribution of its remuneration in comparison with the other distinguished contributions, over which it gained a decisive advantage (the sub-bars for capital in Figure 11 for the period of 2010–2015 are the highest). Thus, the growth was becoming more and more the effect of intensified investment in capital with a decreasing productivity growth and a simultaneous decline in the importance of TFP in the growth of the Polish economy. The situation was therefore not so much worsening, but rather improving at an increasingly slower rate. This is a phenomenon that had already been observed in the world before the financial crisis of 2007–2009 (see Acemoglu, 2003; Klump et al., 2004). This fits in with the logic of the economic growth theory, which postulates that the productivity of capital decreases as its size increases.[109] The phenomenon of a relative saturation with capital occurred in Poland and worldwide.[110]

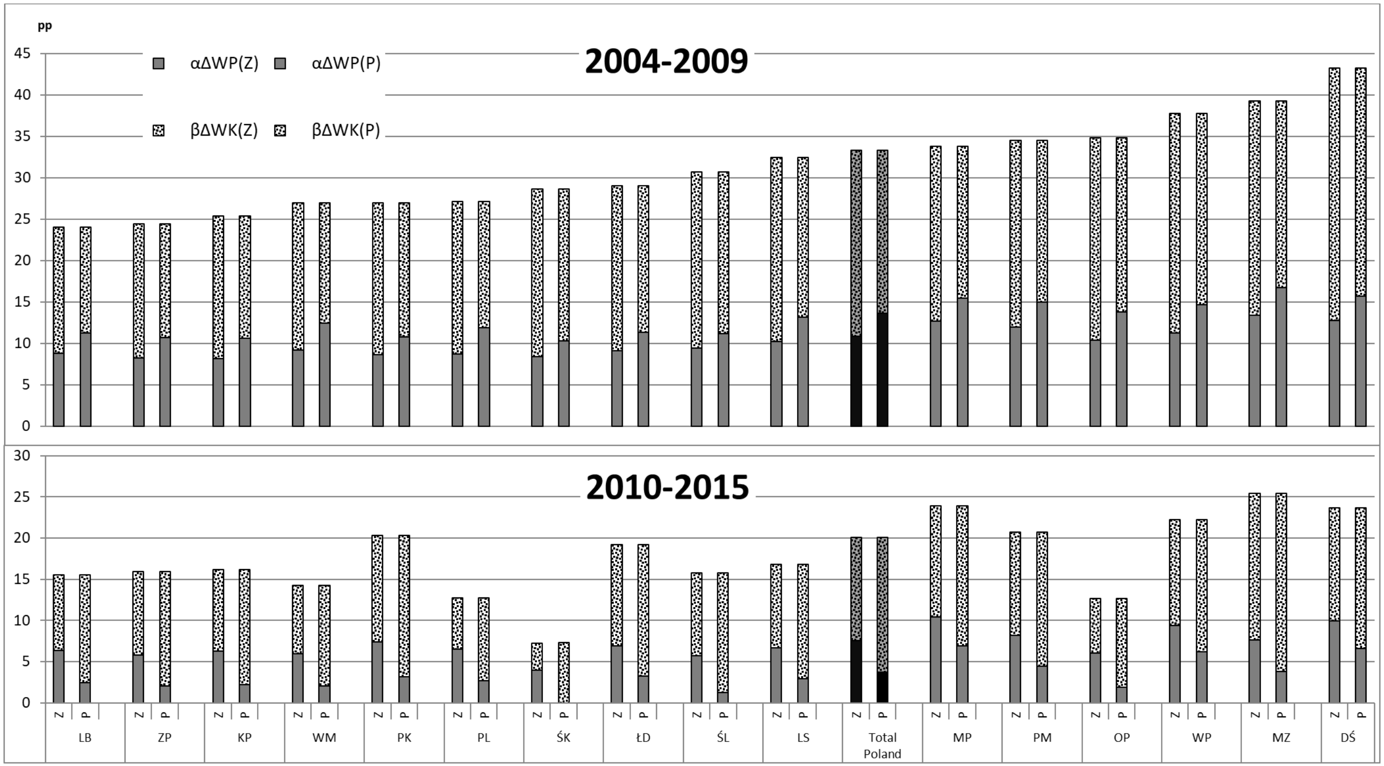
[109] According to different approaches, including *per capita* and relative ones.
[110] In the world, this mainly concerns the emerging economies.

Negative TFP contributions are particularly conspicuous for voivodships such as Świętokrzyskie and Lubuskie in Figure 11 for the period of 2010–2015. In their case, the contribution of the capital stock to the relative growth of GVA (corresponding sub-bars on the right-hand bars) was much higher than the contribution of the remuneration of capital (corresponding sub-bars on the left-hand bars), i.e. the productivity of capital (based on assumptions from the economic theory) not only did not grow in these voivodships, but even declined in the 2010–2015 period. This negative growth of capital productivity is also reflected in the negative contributions of TFP, as indicated by the white sub-bars for these voivodships in Figure 11 for the period of 2010–2015. In the 2004–2009 period, the contribution of capital remuneration to economic growth was greater than the contribution of the capital stock (represented in this study by GVA growth rates) for all voivodships. However, this was no longer the case in the period 2010–2015, which confirms the general conclusion that the importance of capital productivity growth in the economic growth of the country is declining.

All these analyses can be carried out in this way only if the decomposition of the relative growth of GVA into both factor remuneration contributions and factor resources (stocks) contributions and TFP contribution are performed simultaneously and by voivodship. The analysis can, of course, go deeper, since Figure 11 can also be performed for each NACE section or group of NACE sections included in the implemented system of factor decompositions. In this case it means that it is possible to repeat the procedure illustrated in Figure 11 twelve times, i.e. as many times as there are distinguished aggregations at the level of NACE sections in the performed decomposition accounting (there is also a theoretical possibility of performing all these charts for employed persons;, however, as already mentioned, the interpretation of the results is problematic and, since they are similar, the procedure is redundant). The applied methodology also allows for appropriate additional calculations for selected sectors of the economy, being aggregations of NACE sections or groups of sections distinguished in the performed accounting, e.g. the industrial sector can be compared with the service sector; many other compilations of this kind can be made as well.

The analysis of Figure 12 demonstrates that some additional analytical value results from the decomposition of the relative growth of GVA both for the employed persons and the working persons. The same assumptions have been made here with regard to the results presented on Figure 11, except that both the left- and right-hand bars refer to the decomposition of the relative growth of GVA into remuneration contributions of factors. The left-hand bars correspond to the labour factor considered as employed persons and the right-hand bars to the labour factor considered as working persons.

Figure 12. Comparison of decompositions of compound relative growth of GVA into factor remuneration contributions for employed persons and working persons



Note. Contributions of: $\alpha\Delta WP(Z)$ – labour remuneration for employed persons (Z), $\beta\Delta WK(Z)$ – capital remuneration for employed persons (Z), $\alpha\Delta WP(P)$ – labour remuneration for working persons (P), $\beta\Delta WK(P)$ – capital remuneration for working persons (P). Voivodships as in Figure 11.

Source: Kotlewski (2019).

Figure 12 shows that for all voivodships in the period of 2004–2009, i.e. for the entire country, the contribution of the remuneration of the working persons to the economic growth understood as a relative increase in GVA is greater than the contribution of the remuneration of the employed persons. In part, this situation results from the fact that the weight parameter, i.e. the share of labour usually denoted as α (with appropriate indices) is simply larger for the working persons, because the share of the employed persons together with the self-employed, i.e. the working persons in GVA is larger than the share of the employed persons alone. The distribution of this difference between voivodships can potentially be the subject of a detailed analysis.

Much more significant results were obtained for the 2010–2015 period, especially when comparing them to the previous period of 2004–2009. They indicate that the contribution of the remuneration of the working persons to economic growth (understood as a relative increase in GVA, as above) in this period was lower than the contribution of the remuneration of the employed persons, and this proves that the contribution of the remuneration of the self-employed to economic growth was negative in the 2010–2015 period. Thus, the situation from the period 2004–2009, when the self-employed significantly contributed to economic growth, was reversed.

These observations would also be feasible through charts produced for the employed and the working persons, with a TFP extraction, but they would be less clear, which is why they are not quoted in the said version. Moreover, the already mentioned interpretation problem would also occur in relation to the decomposition with an extraction of the TFP for the employed persons.

An attempt can be made to interpret this result as follows: in the period 2004–2009, the self-employed played a significant role in economic growth, as small, often one-person businesses thrived. This is particularly true for the agricultural sector and numerous services, especially tourism, catering, legal services, etc. Therefore, one can see that the difference between the remuneration contribution of the working persons and the remuneration contribution of the employed persons is particularly great for Wielkopolskie Voivodship (since agriculture plays a significant role in this region) and for Warmińsko-Mazurskie Voivodship (a major role played by tourism and small catering). This difference is also more considerable for Mazowieckie Voivodship (where legal services, small financial intermediation, etc. play a dominant role) in relation to Dolnośląskie Voivodship – the two growth leaders in both periods. In turn, the period of 2010–2015 saw a consolidation of these services, including their acquisition by larger, more specialised firms. This may have been accompanied by a reduction in the 'chimneys' remuneration (very high, outlier remuneration rates) for the self-employed due to increased competition in an already more mature market and the elimination of windfall profits.

Both Figure 11 and 12 demonstrate an increase in the importance of the contribution of capital and its remuneration to economic growth in the 2010–2015 period. This observation is consistent with the general observations for the world in relation to emerging markets (van Ark, 2016). In the case of the Polish economy, it can be partly associated with the country's access to EU funds, which stimulate the growth of investments, e.g. infrastructure investments. In turn, infrastructure investments are being effectively transformed into economic growth in the very long run, i.e. their contribution is already great, but the resulting increase in the rate of the relative GVA growth is presently still minor. Capital stock and capital remuneration contributions should therefore be increased in the 2010–2015 period in relation to the 2004–2009 period, although capital stock contributions to a relatively greater extent.

All these observations become clearer and more informative when the decomposition is performed according to two dichotomies, i.e. both for the factor remuneration contributions and for factor resources (stocks) and TFP contributions, and also for both the employed persons and for the working persons. Similarly to Figure 11, Figure 12 can be manifolded 12 times, i.e. for each A12 aggregation occurring in this decomposition accounting. In principle, the presented analytical digression goes beyond the strict content-related scope of this paper, which is mainly oriented towards the methodology of decompositions performed within the framework of the research projects carried out and implemented in Statistics Poland, and only serves to demonstrate the sense of the undertaken methodological work and the purposefulness of the performed accounting.[111]

[111] The theoretical development of the principles of carrying out productivity accounting could, in the future, cover all levels of economic activity, including the global economy – both at the level of individuals, enterprises, regions (e.g. communities, districts and provinces), as well as countries, world regions and the world as a whole. Therefore, it seems that it is necessary to strive for an increasingly better understanding of the mechanisms of the interdependence between these elements. More on the subject can be found in Sulmicki's work (1977) which is entirely devoted to the systemic principles of international economy management; in the sectoral dimension (transport system), reference can be made to Brdulak's (1989) work.

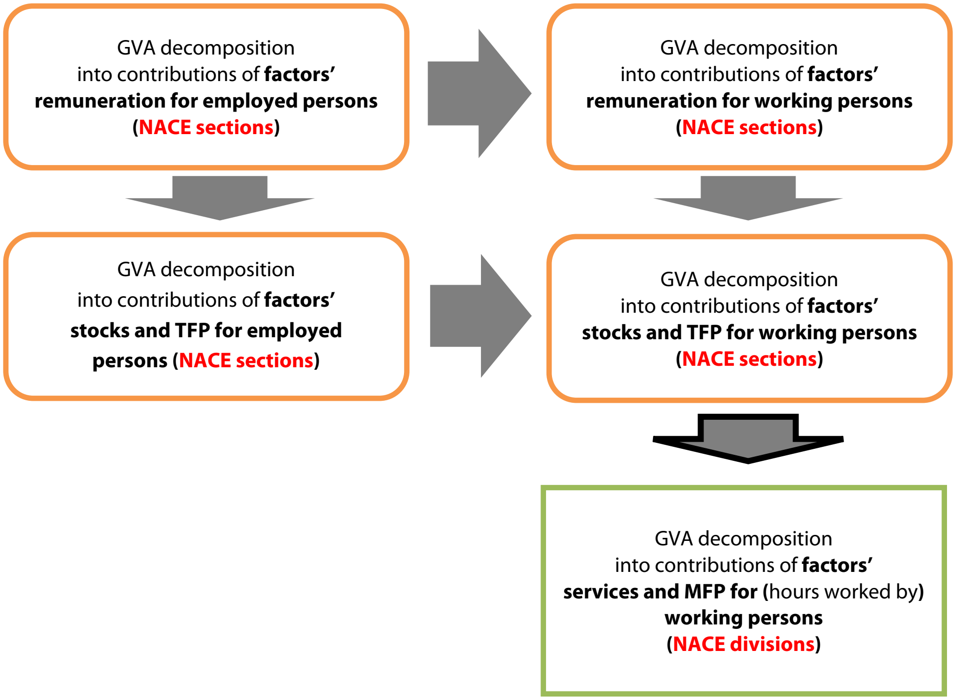
Chapter 4
Regional KLEMS productivity accounting

In Chapter 4, we will present the methodological foundations of the presently developed KLEMS productivity accounting for the Polish economy by voivodship, the results of which will be published after the calculations have been finished. This kind of accounting is much more demanding from the point of view of input data for the calculations than the previously performed types of regional factor decomposition accounting presented in Chapter 3. However, they can be referred to in the methodological work on the regional KLEMS productivity accounting, because the latter is in fact a methodological development of the already performed variant of the Solow decomposition accounting. KLEMS productivity accounting theoretically makes it possible to refine and deepen the factor analysis of economic growth in the way already outlined in general by the Solow-type decomposition. This is its additional advantage, which, however, does not diminish specific analytical advantages of performing factor decomposition in a multi-variant manner, as presented in the previous chapter.

4.1. KLEMS-type decomposition as a development of Solow-type decomposition

As indicated in the introduction to this chapter, KLEMS regional productivity accounting by voivodship can be treated as a specific development of the essentially simpler Solow-type decomposition. Only a few of its characteristics are different from the latter, but these are of considerable analytical importance. Moreover, these additional calculations are very labour-intensive. In order to give a general picture of activities related to the methodological work on KLEMS regional productivity accounting, Figure 13 juxtaposes it with different types of regional multi-variant factor decomposition accounting, previously carried out within the framework of the TAOP II and presented in Chapter 3.

Figure 13. Regional KLEMS productivity accounting juxtaposed with different types of regional factor decomposition accounting performed within the framework of TAOP II



Source: author's work.

Figure 13 shows that KLEMS regional productivity accounting can be treated as a development of only one variant of the four-variant factor decomposition, so its performance by voivodship allows the deepening of the decomposition of this variant, but at the expense of losing the possibility of carrying out a multi-variant decomposition. However, in the light of the supply-side economic theory, this variant is the most relevant factor representation of the processes of economic growth of all the four options carried out within the framework of TAOP II.

A KLEMS-type decomposition cannot be performed for factor remuneration in both the employed persons' and working persons' versions, since TFP is not extracted in these versions. Therefore, there is also no basis for extracting the variant of this variable applicable in KLEMS productivity accounting in the form of multi-factor productivity (MFP). The factor decomposition performed in the framework of TAOP II, on the other hand, where TFP is extracted for employed persons, poses some interpretation problems (which was explained in Chapter 3). In this chapter, we will discuss the differences between the above-mentioned factor decomposition in one of the four variants (i.e. of the Solow type) into factor resources (stocks) and

TFP contributions, and the factor decomposition implemented within the frameworks of KLEMS regional productivity accounting.

The factor decomposition accounting presented in Chapter 3 was performed at the aggregation level of groups of sections and sections of NACE (A12 aggregations), while KLEMS productivity accounting is performed at the aggregation level of groups of divisions and divisions of NACE (A34 aggregations). In other words, instead of a division into the 12 lowest aggregations (by NACE activity) and the 'Total' category, used in the factor decomposition, a division into 34 lowest aggregations was used, although also according to the NACE activities. These 34 KLEMS aggregations are complemented by 13 higher aggregations, i.e. groups of sections and sections of NACE, traditionally used e.g. on the EU KLEMS platform, and by two macroeconomic aggregations for the whole aggregate economy ('Total') and for the 'market economy', customarily defined in KLEMS productivity accounting as the aggregate economy without sections L, O, P and Q. However, these differences have no other consequences for the interpretation of results based on both types of the decomposition than the possibility of making the analysis more detailed.

Important differences between the two types of decomposition considered, affecting the calculation technique to some extent, include understanding the labour factor not as the number of working persons, but as the number of hours worked by the working persons. The rates of change in these values differ slightly, but relatively more so for the self-employed than for the employed persons. When adjusting the figures for the employed persons by the number of the self-employed to the figures for the working persons, noticeable differences will therefore occur. They are also visible in the graphs, i.e. they cannot be interpreted as mere tool-associated deviations. At the same time, these differences do not lead to completely contradictory results for the analysis. Compared to the results of the KLEMS-type decomposition, the results obtained by the simpler factor decomposition should be interpreted more cautiously and the conclusions should be presented in a less definite manner.

A more discrete approach in KLEMS regional productivity accounting implies an increased demand for statistical data, more detailed than the data needed to implement the simpler factor decompositions. Moreover, certain data had to be estimated accordingly, i.e. additional calculations had to be performed, which are described in the further sub-sections.

Similarly as at the level of the aggregate Polish economy, the initial phase of the regional KLEMS productivity accounting implementation involves, as mentioned in Chapter 2, the decomposition of the relative growth of GVA, i.e. the core of the decomposition accounting implemented within the framework of KLEMS productivity accounting, which carries the most important information on the condition of the economy. Potential developments presented in Chapter 2, consisting in the additional decomposition of the contribution of the labour factor and the decomposition of the relative growth of gross output, will be possible depending on the availability of relevant data.

The most significant difference in the light of the theory, however, are different definitions of factor inputs. KLEMS productivity accounting defines these not as factor remuneration contributions or contributions of their resources (stocks) and TFP, but as the contributions of factor services (which should be understood in line with Chapters 1 and 2 and what will be presented below), and as the MFP contribution instead of the TFP contribution.

4.2. Calculations related to labour services

Instead of the more traditional value used in regional factor decomposition accounting (Chapter 3), i.e. the resource (stock) of the labour factor, what should be used in KLEMS regional productivity accounting is a value referred to as labour factor services. However, the data on the resources (stocks) of the labour factor at the lowest adopted aggregations still remain necessary to calculate the values of labour factor services, which are directly unobservable. What is important and similar to what happens in the case of aggregations at the level of sections and divisions of NACE (so lower than the macroeconomic level), is that the difference in calculation results at the level of regional aggregations (also lower than macroeconomic aggregations) should be visible to a greater extent than at the macroeconomic level (i.e. at the level of the whole Polish economy and the whole market economy in Poland). This might influence significantly comparative analyses made at the level of NACE economic sectors and at the level of selected regions of the country. In addition, it is important that the resource of the labour factor for calculating the services of this factor used in KLEMS productivity accounting is the number of hours worked by working persons, not the number of working persons.

In KLEMS productivity accounting,[112] the labour factor at the lowest aggregation is broken down into the contributions of different types of labour. It is assumed that there are 18 types of work, which is the result of its division into two sexes, three age groups and three levels of education (this is explained in more detail in Chapter 1). The types of work are aggregated using the Törnqvist quantity index, thanks to which a value is obtained that, in theory, represents the services of the labour factor. In KLEMS regional productivity accounting, this concept of labour factor services should also be implemented at the level of selected regional aggregations, which in Poland's case is the level of voivodships.

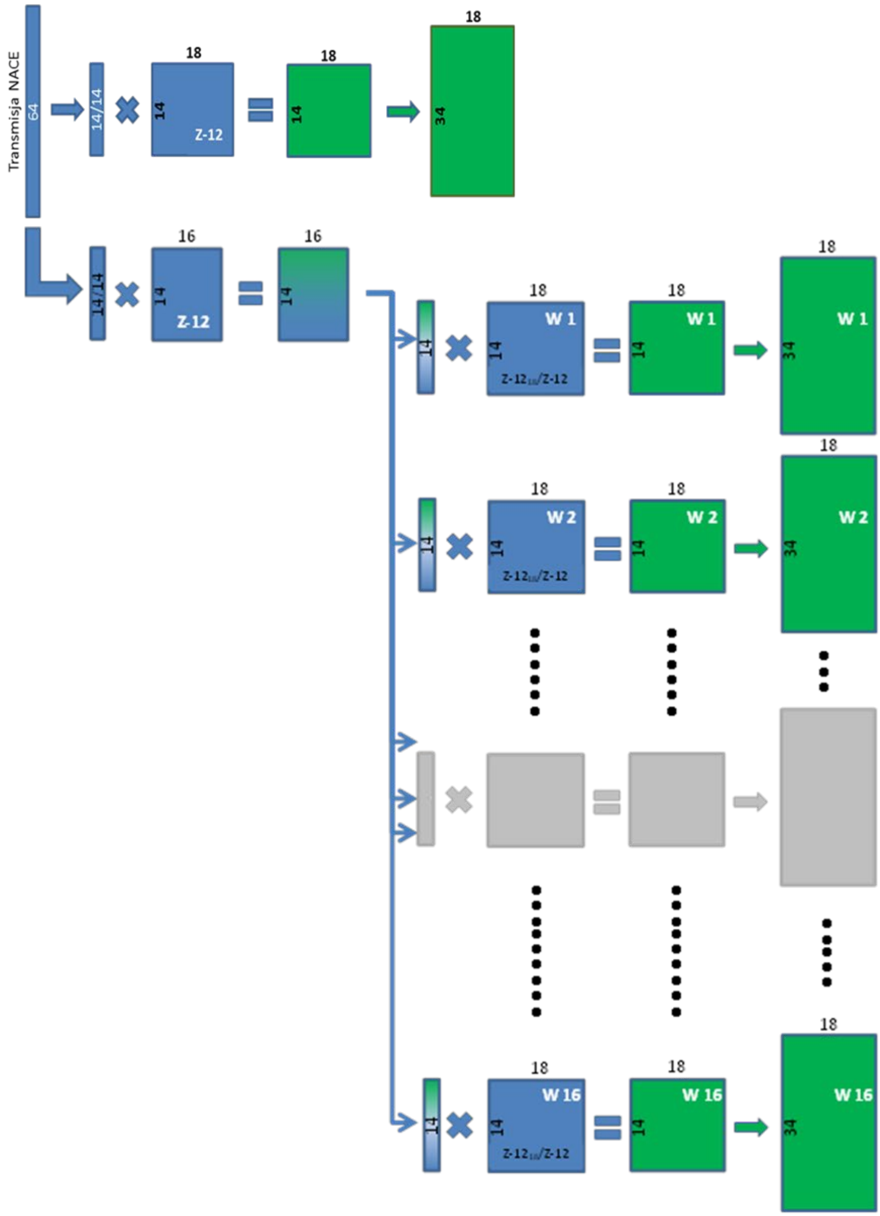
[112] Some other decomposition methodologies have experimented with divisions of the labour factor into some other types.

Referring to the difference between KLEMS productivity accounting and the classical Solow-type decomposition, it should be said that the results of accounting performed by means of these two methods should be similar to some extent, especially at the level of macroeconomic aggregations, which in a way confirms the validity of the calculations performed and the concepts on which they are based. In KLEMS productivity accounting, an attempt to improve the quality of different types of growth decomposition accounting by increasing their precision and the conceptual consistency with the theory has been made. However, this led to increased requirements as regards the availability of appropriate statistical data (which, if not of adequate quality, would not give KLEMS productivity accounting any significant advantage over the classical Solow-type decomposition). Therefore, the latter usually can be done before the KLEMS productivity accounting decomposition, or sometimes it is the only possible option – when appropriate input data for KLEMS-type decomposition accounting is not available. In addition, different kinds of simpler factor decomposition accounting can be implemented in a multi-variant version, including the Solow type, which offers additional analytical advantages.

Similarly to the aggregate level for the entire economy (which was described in Chapter 2), the basic structure which allows the calculation of labour factor services at the level of voivodship aggregations comes from the Z-12 survey (the results of this survey by voivodship have only recently been made available through Statistics Poland, thanks to the fact that additional work was carried out on statistical data dedicated to KLEMS productivity accounting). This structure (by voivodship) should be used to distribute the data covering the entire labour market in individual voivodships according to sub-aggregations into 18 types of labour, at the level of each of the 34 lowest sectors (A34 aggregations) distinguished in KLEMS productivity accounting, and additionally 13 intermediate aggregations at the section level and two macroeconomic aggregations for the aggregate economy and for the market economy.

Figure 14 in the top left-hand-side corner shows the technique of adjusting data, illustrated earlier in Figure 3 – this technique should still be used to produce accounting for the whole Polish economy and the market economy (obviously according to the 34 lowest KLEMS aggregations together with other intermediate and macroeconomic ones). The diagram below shows how this technique has been modified to decompose data available for the whole labour market and the whole Polish economy, taken from TTs into data by 16 voivodships and by 18 types of labour. As can be seen, data need to be first distributed by 16 voivodships before they are distributed by 18 types of labour, and the latter distribution has to be repeated 16 times – for each voivodship.

Figure 14. Preparation of data for the labour production factor by voivodship



Source: author's work.

The number 14, which marks the height of the rectangles in the diagram, symbolises 14 KLEMS aggregations (A14 aggregations), common for both the NACE 1 and NACE 2 classification systems. In the situation where it was possible to obtain data fully converted to the NACE 2 classification system for the whole period for which KLEMS productivity accounting has been performed, i.e. since 2005, the number 14 would have to be replaced by 34 for all the rectangles of the height of 14, and the last data conversion phases (symbolised in Figure 14 by the transformation of green rectangles of the 14 × 18 size into green rectangles of the 34 × 18 size) would have to be removed from this diagram.

The estimation of data by voivodship starts with the calculation according to equation (32), which is a part of the following system of equations:

$$\begin{aligned} H_{A14\epsilon TT} &= \textstyle\sum_{(A64\epsilon TT)\epsilon(A14\epsilon TT)} H_{A64\epsilon TT}, \\ H_{A14\epsilon TT,W} &= \frac{H_{A14\epsilon TT}}{H_{A14\epsilon(Z-12)}} H_{A14\epsilon(Z-12),W}. \end{aligned} \tag{83}$$

The above makes it possible to create a vertical vector consisting of the $H_{A14\epsilon TT}/H_{A14\epsilon(Z-12)}$ ratios, mentioned in Chapter 2, where $H_{A14\epsilon TT}$ is the number of hours worked in A14 aggregations created for data from TTs and $H_{A14\epsilon(Z-12)}$ is the number of hours worked in A14 aggregations created for data from Z-12 sample survey. This vector serves to adjust the $H_{A14\epsilon(Z-12),W}$ aggregations, i.e. the number of hours worked in the A14 aggregations created for the Z-12 sample survey data, by voivodship W. Each of the resulting $H_{A14\epsilon TT,W}$ data vectors, already suitable for KLEMS productivity accounting in terms of the value for each voivodship W (vertical narrow rectangles marked with the number 14 in Figure 14), then has to be decomposed by type of work l:

$$H_{A14\epsilon KLEMS,W,l} = H_{A14\epsilon TT,W} \frac{H_{A14\epsilon(Z-12),W,l}}{H_{A14\epsilon(Z-12),W}}. \tag{84}$$

In the above equation, this was done by multiplying the $H_{A14\epsilon TT,W}$ vectors, different for each voivodship W, by a table matrix of a height equal to the number of A14 aggregations, created from the ratios of the number of hours worked in types of labour l on the basis of data from Z-12 sample survey by voivodship W to the number of hours worked for all types of labour in total by voivodship W. These array matrices are separate for each voivodship, hence the above calculation is necessary – it is not enough to distribute the final data for the aggregate economy (the last rectangle of the 14 × 18 dimensions in Figure 3, also present in the upper part of Figure 14) by the $H_{A14\epsilon TT,W}$ vectors. Once this has been performed, what is left to be done is to distribute the A14 aggregations already corresponding in their values to the needs of KLEMS productivity accounting into the A34 aggregations used in this accounting:

$$H_{A34\epsilon KLEMS,W,l} = \frac{H_{(A34\epsilon TT)\epsilon(A14\epsilon TT),W}}{H_{A14\epsilon TT,W}} H_{A14\epsilon KLEMS,W,l}. \quad (85)$$

This equation distributes data for A14 aggregations, corresponding – from the point of view of their values – to the needs of the KLEMS productivity accounting by voivodship W and by type of work l, into lower A34 aggregations used in the KLEMS productivity accounting, also by voivodship W and type of work l. For this purpose, equation (85) uses vectors consisting of the ratios of the number of hours worked in aggregations A34 created for voivodships W from data from TTs, belonging to the respective A14 aggregations also created from data from the TTs, to the number of hours worked in A14 aggregations created from data from the TT tables for voivodships W (which appears on the left-hand side of the lower equations in the system of equations (83)).

If in the future the necessary data from the Z-12 survey becomes fully available in the NACE rev. 2 classification, the use of A14 aggregations, common for both the NACE 1 and NACE 2 systems, will become redundant, and equations (83) will take the form:

$$\begin{aligned} H_{A34\epsilon TT} &= \textstyle\sum_{(A64\epsilon TT)\epsilon(A34\epsilon TT)} H_{A64\epsilon TT}, \\ H_{A34\epsilon TT,W} &= \frac{H_{A34\epsilon TT}}{H_{A34\epsilon(Z-12)}} H_{A34\epsilon(Z-12),W}, \end{aligned} \quad (86)$$

where the A14 aggregations have been replaced by the A34 aggregations. In turn, equation (84) would then need to be replaced by the following equation:

$$H_{A34\epsilon KLEMS,W,l} = H_{A34\epsilon TT,W} \frac{H_{A34\epsilon(Z-12),W,l}}{H_{A34\epsilon(Z-12),W}}, \quad (87)$$

and equation (85) would then become redundant.

The answer to the question about data availability in the NACE 2 classification system for the whole period for which KLEMS productivity accounting has been performed is, however, open. It is very likely that the data coverage in the NACE 2 system will be incomplete at some stage of the accounting, which will require the adoption of a hybrid solution. In such a case, the KLEMS disaggregation into 14 aggregations could serve as a methodological ‘reserve tool’ of a potential usability in the situation where only incomplete data is available for particular calculations of a developmental character under the NACE 2 system.

Similarly as in the calculations for the aggregate Polish economy presented in Chapter 2, the labour factor should be calculated according to this method taking into account self-employment, both for the number of hours worked and for the remuneration of this factor, onto which the proportions for the number of hours worked have been transferred at the level of each respective sectoral aggregation required in KLEMS productivity accounting and additionally for each voivodship. The data prepared in this way have to be inserted into equations (16) and (17), and the calculations have to be repeated 16 times, for each of the 16 voivodships. Thus, at the level of voivodships, not only the contribution of the labour factor services is calculated as above, but also the contribution of the labour factor resource in the form of the contribution of hours worked, and the contribution of the labour quality (or labour composition) at the level of voivodships as well.

All other issues related to the labour factor have been described in Chapters 1 and 2. Again, it will be possible to perform regional KLEMS productivity accounting according to four variants (denoted by the letters A, B, C and D, or A', B', C' and D'), as a consequence of two dichotomies in the accounting, the first of which results from two different ways of understanding the labour quality (as presented in Figure 4), and the other from the fact that the capital category may or may not include the resident capital (as described in the commentary to Figure 4). What is more, it will be possible to perform a deepened decomposition of the contribution of the labour factor by voivodship in the further development of KLEMS regional productivity accounting, as presented in Figure 7.

The latter possibility results from the fact that the Z-12 survey, available since 2020, which is the main source of data structure, offers data on the number of employed persons, the number of hours worked by employed persons during the year and the average hourly remuneration of employed persons during the year (by NACE section and 18 types of work) classified according to the voivodship division as well as for the whole Polish economy (see Chapter 2). In the methodological basis of KLEMS, it is assumed that this kind of accounting should use, if possible throughout its whole course, the proportions of hours worked for the distributions of 'true' (from the point of view of their volume) data prepared in accordance with the SNA as applied e.g. to the TTs including data on the remuneration aggregates of the labour factor. Therefore, the data on hourly wages from the Z-12 survey are not used to adjust the remuneration of employed persons to the remuneration of working persons by the remuneration of the self-employed, but only as a structure for distributing the remuneration by NACE sector, i.e. the 14 KLEMS aggregations (A14 aggregations) common for the NACE 1 and NACE 2 systems, or, if all data were available in the framework of the NACE 2 system, the 34 lowest KLEMS aggregations (A34 aggregations), together with the intermediate aggregations at the section level and the two macroeconomic aggregations, and by the 18 types of labour (the latter being the most important reason for using structures from the Z-12 sample survey). However, the aggregates and structures of hours worked taken from the Z-12 survey are derived by multiplying the number of employed persons by the number of hours worked per employed person, so the number of employed persons and the number of hours worked per employed person are initially available from the Z-12 survey for the deepened decomposition of the labour factor. This means that there are no significant obstacles to carrying out a deepened decomposition of this factor.

4.3. Calculations related to capital services

In the case of the capital factor, similarly as in the case of the labour factor, KLEMS productivity accounting uses quantities representing not the stock of the factor, but its services (which obviously also applies to KLEMS regional productivity accounting). The contribution of capital services to the relative growth of GVA can theoretically differ from the contribution of the capital stock (i.e., effectively, the contribution of the stock of fixed capital). This difference occurs due to the use of the Törnqvist quantity index for aggregation (this issue is discussed in more detail in Chapters 1 and 2). As in the case of the services of the labour factor, this difference should be relatively greater at the level of individual divisions and sections of NACE, so also at the level of the applied KLEMS aggregations and at the level of voivodship aggregations, than at the level of higher aggregations (including the aggregate level). However, it is smaller than in the case of the labour factor, where at the aggregate level as well there is a noticeable difference between the contribution of labour services and the contribution of the labour resource (stock), interpreted as the contribution of labour quality (which was described in more detail in Chapter 2).

When KLEMS productivity accounting is implemented at the voivodship level, appropriate calculations need to be made to determine the contributions of capital services by voivodship to voivodships' relative GVA growth. This requires access to relevant data for voivodships. These data have to be ultimately divided into different types of fixed assets, according to the 34 lowest KLEMS aggregations (A34 aggregations), intermediate aggregations at the level of NACE 2 sections, and the two macroeconomic aggregations: for the aggregate Polish economy and for the market economy (the latter as defined in KLEMS productivity accounting). At the level of these aggregations, different types of capital (of which, as mentioned, there are seven or eight) should be aggregated using the Törnqvist quantity index, so that the contribution of capital is the contribution of capital services. Again, this method is used because the services of capital themselves are directly empirically unobservable.

Therefore, in this case the basic issue is to distribute by voivodship these data on fixed assets for the aggregate Polish economy that are applicable to the calculations for the whole country (described in Chapter 2), and subsequently to distribute the data obtained for voivodships by seven or eight types of fixed assets. Data by voivodship are available in evidentiary (register) prices (in other words, historical prices), so they can only be used as a structure for the distribution of data considered as 'true', i.e. suitable for calculations within the framework of KLEMS productivity accounting, available for the aggregate Polish economy, and not directly. Since these data at register prices are available in the NACE 1 and NACE 2 classification systems, it is necessary to use the A14 aggregations common to these systems used in KLEMS productivity accounting.

The procedure of capital services distribution by voivodship is thus similar (although not identical) to that shown in Figure 14 for the labour factor. The upper, smaller part of this diagram can in this case be removed, as the structure of register (historical) prices is not used for data distribution at the aggregate economy level (in contrast to the labour factor, for which the Z-12 sample survey data are also used for the aggregate economy estimates), as the 'true', i.e. relevant data that were collected empirically are available. In the lower, larger part of the diagram (Figure 14), showing estimates by voivodship, some changes should be made. The 14×16 rectangle, symbolising the division into the common A14 aggregates used in KLEMS productivity calculation, according to 16 voivodships, marked with the Z-12 symbol, should be marked with a symbol indicating that these are data in register prices, e.g. with symbol E (from evidentiary). It should be remembered that the data in this case concern fixed assets and not the number of hours worked. In other rectangles where the Z-12 symbol appears, it should also be replaced by symbol E. This also applies to the relationships marked by the Z-12_{18}/Z-12 symbol, which should be replaced by E_7/E or E_8/E. The dimension of the 14×18 rectangles for the consecutive voivodships marked with symbols $W1$, $W2$ … and $W16$ should be replaced with the 14×7 dimension (without the resident capital) or the 14×8 dimension (including the resident capital).

Here, the methodology adopted for data distribution in KLEMS productivity accounting meets the methodology of data distribution in the factor decomposition presented in Chapter 3. More specifically, a certain simplifying assumption is made, according to which the distribution of data within each section by the above-mentioned types of capital is the same for each voivodship. This means that the rectangles already mentioned, of the 14×7 or 14×8 dimensions, are identical for each voivodship, and the differences between voivodships in this respect result solely from a different sectoral structure, i.e. a different vector of dimension 14 (A14 aggregations), by which the rectangle common to all voivodships should be multiplied. Hence the conclusion that in equation (84), apart from the changes in the subscripts that make it relevant to the distribution of capital services, subscript W indicating voivodships should also be removed in two out of four cases.

In order to present these accounting operations more clearly, the transformed equations (83)–(85) read as follows:

$$K_{A14\epsilon RN} = \sum_{(A88\epsilon RN)\epsilon(A14\epsilon RN)} K_{A88\epsilon RN},$$
$$K_{A14\epsilon RN,W} = \frac{K_{A14\epsilon RN}}{K_{A14\epsilon E}} K_{A14\epsilon E,W}. \tag{88}$$

In these equations, analogous to equations (83) for the labour factor, the number of hours worked H has been replaced by the stock of fixed capital K, the symbol indicating that the Z-12 sample survey has been replaced by the symbol indicating register prices E, and the TT symbol for the data transmitted to Eurostat has been replaced by the symbol indicating the data taken from the National Accounts RN. The original National Accounts data are broken down into A88 aggregations, so their symbol has replaced the symbol for the A64 aggregations required by Eurostat in the TTs. Equation (84)should be subsequently replaced by the following:

$$K_{A14\epsilon KLEMS,W,k} = K_{A14\epsilon RN,W} \frac{K_{A14\epsilon E,k}}{K_{A14\epsilon E}}. \tag{89}$$

In this equation, analogous to equation (84) for the labour factor, the number of hours worked H has been replaced by the stock of fixed capital K, the symbol indicating the Z-12 sample survey has been replaced by the symbol indicating registered prices E, the TT symbol for data reported to Eurostat has been replaced by the RN symbol indicating the data from the National Accounts, and the l symbol indicating the 18 types of labour has been replaced by the k symbol indicating seven or eight types of capital. As mentioned before, the W symbol indicating the differentiation by 16 Polish voivodships has also been removed from two places, as necessitated by an accounting simplification. Consequently, equation (85) should be replaced by the following:

$$K_{A34\epsilon KLEMS,W,k} = \frac{K_{(A34\epsilon RN)\epsilon(A14\epsilon RN),W}}{K_{A14\epsilon RN,W}} K_{A14\epsilon KLEMS,W,k}, \tag{90}$$

where the symbols have been appropriately replaced as explained above.

In this way, data by voivodship can be obtained for the capital factor, suitable for KLEMS productivity accounting. These data are slightly less accurate due to the use of simplifications, but nevertheless they comply with the standards of the accounting. Moreover, lower accuracy in the capital-related types of accounting is common for all the performers of KLEMS productivity accounting. Unfortunately, although it seems possible to obtain data for the labour factor entirely according to the NACE 2 classification (thanks to which it would be possible to use only the target A34 aggregations together with intermediate and macroeconomic aggregations instead of the A14 aggregations), such a possibility should not be expected in the near future for the ‘capital’ factor, as no conversion of register prices recorded in the NACE 1 classification system (not necessarily accurate any more) into prices under the NACE 2 classification system is planned. As already mentioned, data for the capital factor for 2008 were collected in register prices under both classification systems. When calculating increments between 2007 and 2008, data under NACE 1 classification system were used, and when calculating those between 2008 and 2009, data under NACE 2 were applied. Such a procedure ensures almost complete consistency of data on charts, which, in turn, proves the efficiency of the method.

A certain problem arises from the need to extract the ICT capital not only at the level of the country’s aggregate economy, but also at the level of individual voivodships. It can be assumed that the ICT capital in each of the A34 aggregations used in KLEMS productivity accounting is utilised to approximately the same degree and in the same proportion to the total capital of that aggregation, regardless of the voivodship. Therefore, the differences between voivodships result from different sectoral structures of their economies. In other words, it can be assumed that the fact the share of the ICT capital in the total capital of particular voivodships varies across them primarily results from the geographical distribution of this capital, determined by the geographical distribution of the individual 34 KLEMS activities, i.e. A34 aggregations. It is assumed that this approximation is better than not extracting the ICT capital at all. The practical observation made in the course of the calculations is that the results obtained using (wherever necessary) the A14 aggregations differ to only a small extent from those obtained using the A34 aggregations.

The possible variant of the above method of distributing data might or might not prove preferable. It consists in assuming the presence of the same proportion for each type of capital from which a given type of ICT capital was extracted, rather than the presence of the same proportion for the whole capital of a given aggregation. It can be thus assumed that of the categories listed in Chapter 2, the following categories of ICT capital: computer equipment (5) and telecommunications equipment (6) are in the same proportion to the category of other machinery and equipment (4) in each of the 34 lowest KLEMS aggregations, regardless of the voivodship. Even if it is necessary to use the 14 ongoing KLEMS aggregations, the assumption of similar intensity of categories 5 and 6 in relation to category 4 in a given sectoral aggregation due to its technological and other specificities seems not far from reality. Similarly, it can be assumed that the computer software category (9) is in the same proportion to the intangible assets category (8) in each of the 34 lowest KLEMS sectoral aggregations, irrespective of the voivodship, even if it is necessary to use the 14 ongoing KLEMS aggregations. This data-distributing variant might prove preferable, but only if it was possible to differentiate between the voivodships' shares in the broader capital categories (4) and (8) in the total capital of a given A34 aggregation, or possibly a given A14 aggregation. However, the differences between this method and the previous, simpler one will be barely noticeable on charts (as indicated by the experience of working with data). In the view of the future development of the accounting and the associated possibility of its future modernisation, it has been decided to perform the calculations in this slightly more subtle way, although at present it still does not matter.

The increments in fixed capital stocks, i.e. the increments in capital stocks, aggregated by the Törnqvist index give a value proportional to the increments in capital services, which, as we know, are not directly observable. Therefore, if the relative increments in capital services themselves (rather than in the unknown absolute levels of capital) are considered, then the problem of determining the contribution of these capital services to the relative increment in GVA is, effectively, solved.

The weights, i.e. the average inter-periodic shares (similarly to the previous shares, in principle calculated by linear interpolation between the shares of the previous year and the current year), are obtained for capital in a residual manner, by assuming that all the weights for the labour and capital factors in the decomposition of the relative growth of GVA sum up to unity. Going a step further, it is even possible to dispense with the linear interpolation procedure there, i.e. to assume that the summation to unity applies to the intertemporal average shares, not to the annual shares from the linear interpolation formulae. The average intertemporal shares of capital remuneration in GVA can therefore be calculated by subtracting the average intertemporal shares of labour remuneration in GVA from the unity. This helps avoiding the formation and accumulation of tool deviations.

4.4. Methods of assessing missing data

Referring to paragraph 3.1.5 which concerns estimation algorithms used in the multi-variant but simplified factor decomposition, it can be assumed that the methods used there should be equally effective in KLEMS regional productivity accounting. There is the same division into 16 voivodships, but also, instead of the A12 aggregations, the A14 aggregations (and often even the A34 aggregations) are used, so the accuracy of the above-mentioned data estimation methods should be even higher.

The rationale for using equation (56) is the same as presented in Chapter 3, but the operations associated with it take the form described in sub-section 4.3. This is a more complex form due to the presence of several A88, A14 and A34 aggregations, whereas in the simplified factor decomposition presented in Chapter 3, only the A12 aggregations (with the possibility of replacing them with the A14 aggregations) were used.

Equation (57) takes the form:

$$KN_{S(A34)W} = \frac{KN_{S(A34)}}{KN_{B(A34)}} KN_{B(A34)W}. \tag{91}$$

It is similar to equation (57), but here it is possible to descend to lower aggregations used in KLEMS productivity accounting. On the basis of relevant experience, it can be presumed that the differences between the voivodships in terms of inflation of capital goods and investment goods result in a large part from a different sectoral composition of these goods, i.e. their different structure by voivodship. However, this differentiation is taken into account in equation (91) to even a greater extent than in equation (57), thanks to the use of the lower A34 aggregations. The deviations from the unknown 'true' values are thus reduced. All other rationale concerning equation (57) described in Chapter 3 apply to equation (91).

The same subscript substitution operation as for equation (57) should be performed for equation (58), which should then take the following form:

$$WDB_{S(A34)W} = \frac{WDB_{S(A34)}}{WDB_{B(A34)}} WDB_{B(A34)W}. \tag{92}$$

In turn, equation (59) takes the form:

$$WP_{S(A34)W} = \frac{WDB_S}{WDB_B} WP_{B(A34)W}. \tag{93}$$

The same arguments apply to equations (92) and (93) as to equations (58) and (59), except that they are more plausible here, since the use of the lower A34 aggregations considerably strengthens the arguments behind them. Of course, this also concerns equation (61), adapted to the form:

$$WK_{S(A34)W} = WDB_{S(A34)W} - WP_{S(A34)W}. \tag{94}$$

The data-adjusting operation described by equation (62) should be replaced by the operations presented in Figure 14 with its accompanying equations. Equation (63) in this situation will take the form:

$$WP_{P(A34)W} = \frac{H_{P(A34)}}{H_{Z(A34)}} WP_{Z(A34)W}, \tag{95}$$

and here the same rationale as that behind the use of the lower A34 aggregations applies, reinforcing the arguments presented in Chapter 3.

In the classical KLEMS productivity accounting, equation (95) is used only at the aggregate level, which could also be done in the case of voivodships. However, the arguments for using the modified equation (64) for the agricultural sector are even stronger here, because there are voivodships in which agriculture plays a greater role than in the others or in the whole country. So, equation (64) should be modified to the following form:

$$WP_{P(A34)W} = \left(WP_{Z(A34)} + DM_{(A34)} \frac{WP_{Z(A34)}}{WDB_{(A34)} - DM_{(A34)}} \right) \frac{WDB_{(A34)W}}{WDB_{(A34)}}. \tag{96}$$

A similar assumption could be made for equation (96), namely that the use of the lower A34 aggregations makes its application preferable to the use of equation (64).

In general, it is assumed in KLEMS regional productivity accounting that since it uses lower aggregations than the multi-variant factor decomposition, the variability of the values associated with the labour and capital factors (which in a large part results from different sectoral compositions of voivodships' economies), will be better reflected in the former rather than the latter.

4.5. Analytical benefits of performing regional KLEMS-type decomposition

The implementation of KLEMS productivity accounting at the regional level has some important advantages over the multi-variant factor decomposition. Not only the concept of the contributions of production factors' services and their sub-decomposition into sub-factor services contributions is important. Also, the use of lower aggregations, in addition to the possibility of a more discrete sectoral analysis, theoretically increases the accuracy of all resulting accounting at higher aggregations. The information on the state of the economy is thus not only more detailed, but also more accurate. This is so despite the fact that the multi-variant factor decomposition has its own special advantage due to a multi-variant shape, as explained in Chapter 3.

Also the performance of the additional methodological and computational work associated with KLEMS productivity accounting at the regional level is justified. Since the performance of a multi-variant KLEMS productivity accounting in a manner similar to the multi-variant factor decomposition is not possible for methodological reasons, both types of decomposition seem to have their own legitimate analytical uses and are complementary to each other.

KLEMS-type decomposition is performed in a number of countries in the world, so the results of this kind of productivity accounting at the regional level are also comparable with those of other countries, even though some of them do not perform their decomposition in the regional aspect (e.g. small countries with sizes similar to Polish voivodships).

Performing a development of KLEMS productivity accounting in the form of a regional decomposition is, as can be seen, possible, but only if all the steps described in the previous chapters have been carried out, and also is quite labour-intensive, as many of these steps need to be performed repeatedly, i.e. for all voivodships. Some automation of these calculations is therefore advisable. Performing the proposed calculations will open up new possibilities for economic analysis and extended interpretation of the results.

Conclusions

KLEMS economic productivity relative growth accounting can undoubtedly be regarded as an important element of mainstream economics, as it derives from the theory of economic growth formulated by Solow (1956). It is also from this theory that Solow's concept of decomposition was developed (1957), which is the basic idea on which KLEMS accounting is based. Solow is a disciple of Leontief (1966), who laid the conceptual foundations for the national accounts statistics. The latter, in turn, became of fundamental substantive importance for KLEMS accounting, and found a direct application in it. However, it is a known fact that the methodologies developed by Leontief could not be fully applied in practice before the advent of the computer era, so they also could not reveal their full substantive value and – as often emphasised – their formal elegance (Domański, 2006, 2012). This also applies, and perhaps to an even greater extent, to KLEMS productivity accounting, the launch of which increased the demand for appropriate statistical data among researchers of economic growth, and, consequently, for tools for their processing. Therefore, the practical implementation of KLEMS accounting was delayed for decades. Previously, the Solow decomposition functioned only as a theoretical artefact, applied very occasionally at the aggregate level of the economy. And this decomposition, too, had to wait a few decades for its modernisation in the form of KLEMS productivity accounting made by Jorgenson and his associates (mainly: Jorgenson et al., 1987, 2005; Jorgenson & Griliches, 1967).

Only as late as in the 21st century, the idea of systematic implementation of KLEMS productivity accounting started being put into effect, as did similar kinds of accounting based on the same theoretical background, such as productivity accounting performed by the OECD. Since 2010, the World KLEMS conferences[113] have been organised systematically every two years. The sixth World KLEMS conference was to be held in the year of the publication of this monograph, but was postponed due to the outbreak of the COVID-19 pandemic. The KLEMS productivity accounting result data was first published in 2007 by the EU KLEMS consortium under the auspices of the University of Groningen. This first edition was very ambitious, as it covered the entire EU (then 25 countries) plus Japan and the United States of America. The 2009 edition added Australia, Canada and South Korea. However, in the same period the number of countries which performed an economic growth decomposition, i.e. the essential part of KLEMS accounting, decreased. This happened due to methodological difficulties and too much of initial optimism in the first edition. For Poland as well, this decomposition was carried out only in the first edition. The development of the EU KLEMS platform encountered an obstacle in the

[113] These conferences resulted in the publication of Jorgenson et al. (2016).

form of problems with obtaining appropriate statistical data for KLEMS productivity accounting. These problems continue to this day.

The above-mentioned high requirements for statistical data, inherent in KLEMS productivity accounting, are, moreover, the reason for its limited use. Rather than being an important part of the mainstream economics, its performance is frequently still a niche phenomenon, and its development is proceeding with a considerable delay in relation to the basic, recognised and well-established theory that underpins it. The great value of KLEMS productivity accounting as the emanation of the common understanding of economic processes, and the simultaneous frequent inability to perform it on the basis of directly available statistical data, are together the main reasons for undertaking research work aimed at its implementation. This work, of an innovative character, should focus on statistical data for the Polish economy, as in Poland the methodology for KLEMS productivity accounting is less known than in the developed Western countries. Even there, however, this type of research seems to be fairly innovative.

The result of the research work undertaken in Statistics Poland was the gradual implementation of KLEMS productivity accounting for the Polish economy. In addition to the implementation of the basic decomposition of the relative GVA growth into the contributions of factors' services and MFP, an additional, deeper decomposition of the labour contribution was carried out, as was the decomposition of gross output. This additional decomposition of the labour factor contribution seems to have allowed some insight into the causes of the mild course of the 2007–2009 global financial crisis for the Polish economy. The decomposition of the relative gross output growth together with the more commonly performed decomposition of the relative GVA growth may, on the other hand, serve as a litmus test for certain changes in the economy, mainly related to the development of outsourcing and leasing and their possible macroeconomic effects.

However, when it comes to the strictly scientific side of the growth decomposition accounting, the most innovative part is its implementation at the regional level (by voivodship). Not many countries have undertaken this task – from among other European countries, only Spain has done it. The specific feature of the regional decomposition accounting for the Polish economy presented in this monograph is the gradual approach. According to it, a multi-variant factor decomposition was carried out as first, and it turned out that thanks to its specificity, it was possible to gain an additional insight into the economic processes related to the observation of self-employment and the factor origin of TFP growth, which also made it possible to see the macro- and the meso-economic effects of these phenomena. It also turned out that the methodology for a multi-variant factor decomposition, after having been further developed, allows the subsequent performance of KLEMS regional productivity accounting and the determination of more analytically-discrete factor contributions and MFP contribution, and its performance by division into lower aggregations at the NACE section level than in the case of the simpler version of factor decomposition. Its results will be published soon after this monograph. The implementation of KLEMS productivity accounting, as well as other previously-mentioned kinds of factor decomposition accounting at the regional level, bridge the gap between macroeconomics and regional science (as of Isard, 1960), i.e. on the one hand, the economic geography is supplemented by sound quantitative studies, and on the other, the application scope of basic macroeconomics is increasing.

What is the use of KLEMS productivity accounting? At first glance, it serves mainly the ex-post analyses of observed economic processes. It thus has an explanatory function, and its potential here is even greater than one might expect. Despite the fact that KLEMS productivity accounting (as well as other types of economic growth decomposition accounting) was developed as an emanation of and in response to the needs of the supply-side theory of economics, thanks to its objective way of dealing with statistical data, it also provides quantitative information that can be effectively used by researchers of different branches of the theory of economics. For example, the data resulting from decomposition accounting, especially from the KLEMS-type accounting, often can be interpreted not only in a way typical for the supply-side economists, but also for the demand-side economists. KLEMS productivity accounting makes therefore a conceptual discourse possible, which is especially in line with the contemporary trend aimed at the synthesis of the demand-side and the supply-side economics.

In this light, the explanatory function of KLEMS productivity accounting may be significantly extended, in such a sense that the recognition of the potential, specific threats might lead to the formulation of certain economic recommendations. These recommendations might even sometimes be of a cutting-edge character (thanks to the observation of certain mechanisms whose macro- and possibly also meso-economic effects could not otherwise be properly assessed). Economic recommendations are usually based on both the supply-side and the demand-side perspectives, as otherwise they would be one-sided and limited to theoretical, rather than practical applications. The latter is the special merit of KLEMS productivity accounting, as in many cases there is no other way to obtain the relevant information than the performance of this kind of accounting or some other, yet similar, kind of decomposition accounting.

Although the results of KLEMS productivity accounting are ex-post results, due to the repeatability of many economic phenomena and the relative perspicacity of the observations made by means of this type of accounting, they can also serve as the basis for formulating forecasts, including very general and scenario-based ones, such as those made in a systematic way by Hillebrand and Closson (2015, pp. 1–25). These results can also be used to identify regional advantages and specialisations, i.e. comparative advantages, and to study the effects of interventions. Looking from another perspective, KLEMS-type decomposition accounting might be used to study the international competitiveness of different countries, especially factor competitiveness (Adamkiewicz, 2019, pp. 179–209; Próchniak, 2019, pp. 231–244). They can also be useful when studying the behaviour of different populations of firms, especially if supplemented with their econometric version, which should be seen as complementary to (rather than competing with) the index-based KLEMS productivity accounting methodology. If phenomena related to economic growth are of non-economic nature (e.g. caused by natural disasters), KLEMS productivity accounting remains a useful tool for the observation of their macroeconomic effects, especially as regards factor contributions.

The performance of KLEMS productivity accounting also enhances the work already done by statisticians, as the data collected are often not used and become outdated for various reasons. Data utilisation in the framework of different types of decomposition accounting can give an appropriate direction to statistical research and data collection campaigns, also from the point of view of their usefulness for the end user, who usually is an economic researcher or a decision-maker responsible for the economic policy at any level of the economic practice.

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